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# Safety Study of TCAS II for Logic Version 6.04

Dr. Michael P. McLaughlin
Dr. Andrew D. Zeitlin

MITRE Corporation
McLean, Virginia



July 1992

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**16. Abstract**

A new System Safety Study of Traffic Alert and Collision Avoidance System II (TCAS II) was performed to compare the safety of logic version 6.04 with the present version 6.0. The study uses a considerable body of encounter data extracted from Automated Radar Terminal System (ARTS) ground-based radar data at eight U.S. sites. Encounter geometries are modeled using the statistics of the observed data. The performance of TCAS logic is simulated using both complete logic versions. The perceived separation statistics are combined with altimetry error models to calculate risk for each encounter geometry. These results are combined in the proportions of encounter geometries found in the airspace at each site. Using a fault tree for the Critical Near Midair Collision event, the Risk Ratio is calculated for each logic version relative to the risk of not using TCAS. This result is discussed in the context of the improved compatibility of the newer logic with respect to the Air Traffic Control (ATC) system, which would increase overall safety.

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Ann Drumm and Ronald Sandholm of MIT Lincoln Laboratory provided information on TCAS surveillance and coordination functions. Thomas Choyce of the Federal Aviation Administration (FAA) Technical Center provided the results of simulation testing for coordinated encounters. The concept of analyzing the simultaneous effects of altimetry error and logic performance was advocated by Dr. Ken Carpenter of the Royal Signals and Radar Establishment (U.K.)

Larry Nivert of FAA Headquarters provided valuable guidance, encouragement, and critical review throughout the project.

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# EXECUTIVE SUMMARY

## INTRODUCTION

The Traffic Alert and Collision Avoidance System II (TCAS II) is becoming widely deployed among the U.S. Air Carrier fleet. The growing body of experience gained using TCAS in daily operations has uncovered certain tendencies in which its Resolution Advisories (RAs) are often judged undesirable and potentially distracting. This Operational experience evolved into new requirements for the TCAS logic which have been addressed by the proposed version 6.04 (v6.04). This Study assesses the safety of TCAS in domestic U.S. airspace. It provides specific comparisons between the previous (version 6.0 [v6.0]) logic and v6.04.

The primary purpose of the current Safety Study is to enable a comparison of the relative safety of the two versions of the logic. The factor of greatest concern is the decreased warning time afforded by TCAS v6.04, which results from threshold reductions designed to eliminate the undesirable advisories.

This study makes use of a significant quantity of data collected from the Automated Radar Terminal System (ARTS) in characterizing aircraft encounter geometries. These data represent an improvement over earlier studies with respect to the validity of the distribution of aircraft geometries in encounters. The study also employs computer modeling and simulation to evaluate the effects and complexities of TCAS logic over a wide variety of situations. While TCAS does not assure separation in every instance, the purpose of the selected approach is to account for many possible situations in their appropriate proportion.

## METHODOLOGY

The principal innovative method underlying much of this analysis is the modeling and simulation of TCAS logic performance for close encounters. The pertinent characteristics of encounter geometries were extracted from radar data, enabling the definition of a set of encounter classes. These were then used in computer simulation to test large numbers of encounters in each class.

Figure ES-1 summarizes this process at a high level.

### Data Sources

MITRE has generated a large database of encounters from aircraft tracks recorded at a number of radar sites throughout the U.S. Eight sites were selected for this study: Burbank (BUR), Coast (CST) (in the Los Angeles Basin), Denver (DEN), Dallas-Fort Worth (DFW), New York (JFK), Minneapolis-Saint Paul (MSP), Seattle (SEA), and Saint Louis

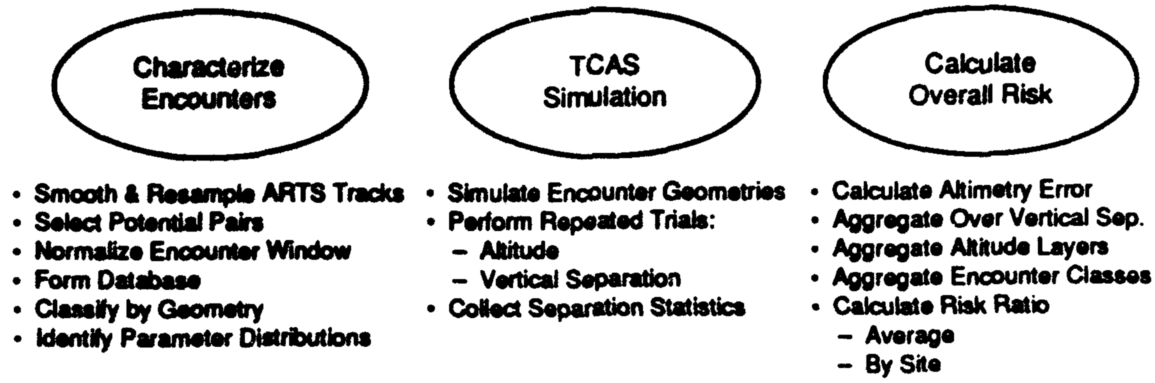



**Figure ES-1. Overview of Risk Calculation**

(STL). These sites were chosen to represent moderate to heavy traffic under differing conditions of geography, traffic type, and Air Traffic Control (ATC) procedure.

### Encounter Statistics

The radar-derived tracks were smoothed and resampled at the 1-second intervals used by TCAS. Candidate pairs were formed and run through the TCAS v6.0 logic to identify those that produced RAs. A database was formed that collected encounter statistics about a 50-second window containing the encounter's point of closest approach (CPA). The encounters were then classified according to the combination of aircraft profiles: level, climbing or descending, or maneuvering to or from level flight.

The number of encounters in each class were counted by site. Also collected by site were the statistics of vertical separation, or Vertical Miss Distance (VMD) at the encounter CPA. Other statistics were collected for the entire set of sites, such as the distribution of vertical rates and accelerations for the various classes.

The encounter classes and distributions form the basis of a theoretical model for aircraft encounters that is based on real data. The modeling enables the assessment of a safety measure and is particularly useful for comparing the relative performance of logic versions.

### Altimetry Error Model

Errors in aircraft altimetry were modeled using the same statistical distributions and variations over absolute altitude as in the previous Safety Studies. However, this study is more comprehensive in examining the effect of this error in combination with TCAS logic performance.

**TCAS Encounter Simulations**

The encounter simulator performs repeated simulations of specified encounter geometries with numerous parameters being varied on successive runs. Every sample encounter is run three times: without TCAS, and with each aircraft in turn carrying and responding to its TCAS, while the other is non-TCAS. The same encounters are rerun for both the v6.0 and v6.04 logic. The simulated TCAS aircraft responds to any TCAS RAs generated according to a model which also provides a range of statistical variation. Separations are compared both with and without TCAS to enable the tabulation of encounter separation statistics.

Encounter simulations are run for one Class (i.e., geometry) at a time. Each set of runs duplicates a Class's geometry for ten "bands" of nominal vertical separation (VMD) without TCAS. To saturate each band, 500 encounters are run in each, with the vertical separation randomly drawn from a uniform distribution. The first band uses vertical separations at CPA from 0 to 100 ft; the second band from 100 to 200 ft; up to the tenth band from 900 to 1000 ft. This study is specifically intended to measure the Risk Ratio. Therefore, only encounters with horizontal miss distance (HMD) less than 500 ft are relevant. The model forms geometries with HMD varying over 0 to 500 ft at the CPA. These encounter Class simulations are repeated for each of the six altitude layers for which TCAS uses different logic parameters.

The output of the simulation represents a distribution of the perceived vertical separation for the encounters. However, the aircraft true altitudes may differ from their reported altitudes. The altimetry error model is used to calculate the probability, averaged over these encounters, that the true separation is less than 100 ft.

**Combining Results for Risk Ratio**

The results of the various simulation runs for the various geometries, vertical separations, and altitude layers are combined to develop an overall ratio of the Near Midair Collision (NMAC) risk with either version of TCAS compared to non-TCAS.

- The results for low-quality and high-quality altimetry are combined according to the proportion of general aviation (GA) aircraft (60 percent) found in the database.
- For each encounter geometry ("Class"), at each of the eight sites, the simulation runs are collected for small bands of vertical separation, without TCAS. The simulation results are combined using the frequencies, from the encounter database, that these separations occur at each site.
- The resulting NMAC probabilities for each Class then are combined in the proportion that the various classes were observed at each site.

- The ratio of this overall NMAC probability is taken between each logic version and no-TCAS.

### Fault Tree Calculations

The result found above is a component of TCAS Risk Ratio that is conditioned upon several events: that the threat is Mode C equipped and tracked, and that the TCAS RA is followed regardless of its correctness. The Fault Tree is used to evaluate these conditions in the correct context.

## RESULTS

### Encounter Classes

Considerable differences are seen in the environments through the distribution of encounter classes represented by this collection of sites. Leveloff encounters are more frequently observed at Dallas than anywhere else. Level encounters predominate in New York and Minneapolis-St Paul. Pairs of descending aircraft are much more common in St. Louis and Denver. Altitude crossings are moderately frequent at Burbank, where more GA mix with air carriers than at these other sites. The differences of this environmental mix also can be seen from figure ES-2, which compares the proportions of some of the larger classes.

### Simulation Results

The basic simulation runs performed for this study (exclusive of degraded surveillance or climb- or descend-inhibited cases) totaled 780,000 encounters for each version of the logic. These runs included all classes with up to one maneuver, which represented 98 percent (1849 of 1889) of the RAs in the database.

The simulations of the various encounter classes at each altitude layer produced vertical separations that were substantially equal to the intended separation. This separation, designated by the value of the logic parameter altitude limit (ALIM), is decreased somewhat in the v6.04 logic.

The greatest number of encounters with poor separation are found in the lowest altitude layer, below 2350 ft. Most of these encounters are unresolved NMACs, rather than induced.

### Combined Altimetry and Logic Performance

The combined effects of logic performance (through perceived separation) and altimetry errors are calculated assuming all RAs are followed. The calculation is done for each of the eight sites using both the encounter class distributions and the vertical separation frequencies within each class as observed at that site. In addition, an average figure is developed, which is based on averaging the class weights and |VMD| across sites.

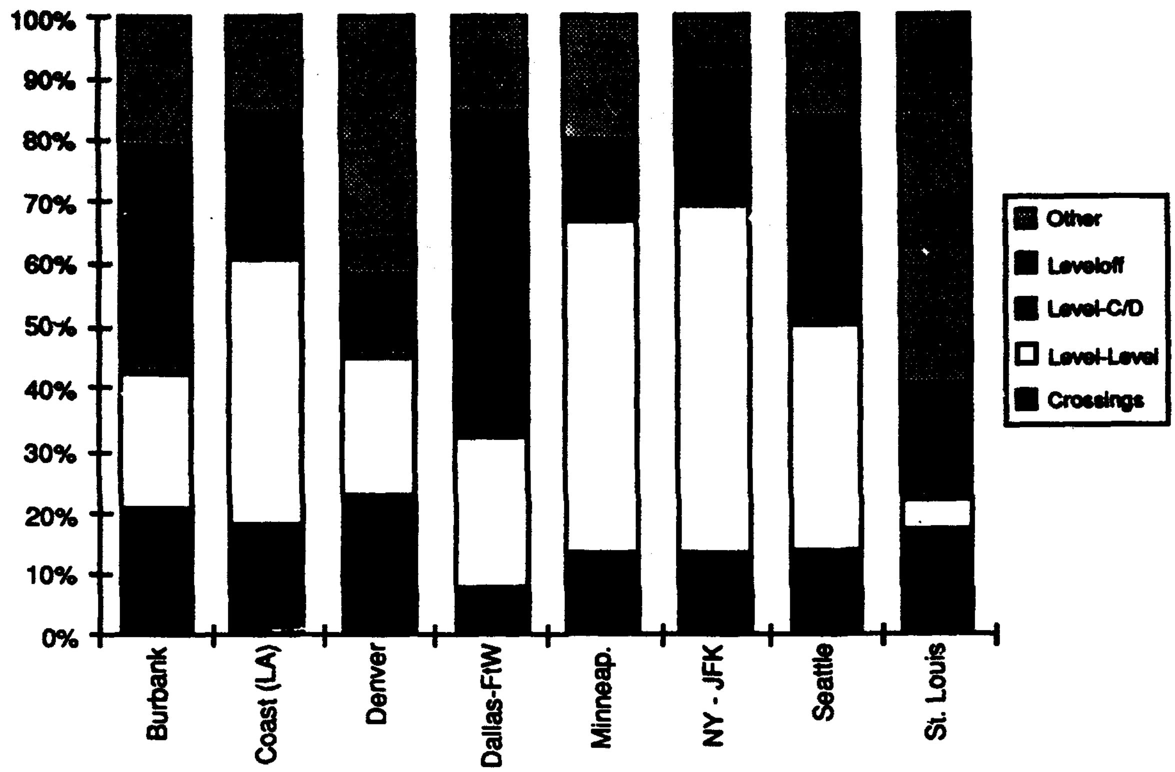



**Figure ES-2. Proportional Frequencies of Encounter Types by Site**

This component of Risk Ratio increases for v6.04 by an amount that is of the same order as the variation of the Risk Ratio among sites for the present v6.0 logic. Risk Ratios are a means of comparison to the NMAC risk prior to TCAS; the incremental values discussed here are 1.5 to 2 orders of magnitude smaller than the no-TCAS risk.

The subsequent Fault Tree calculations make use of this component of Risk.

## Sensitivity Studies

### Imperfect Surveillance

Several encounter geometries were rerun with the probability of TCAS surveillance delivering a Mode C report set to 0.9 and 0.8 respectively. The value 0.8 is thought to be a typical "worst-case" probability of receiving reports through a high density interference environment. For crossing encounters, performance degrades marginally with surveillance quality, but the increment of v6.04 Risk Ratio (versus v6.0) does not change by much (from about one percent to 1.5 percent). For leveloff encounters, the Risk Ratio component actually improves slightly with this marginally degraded surveillance.

These results indicate that imperfect surveillance quality will not have a significant effect on the relative performance of these logic versions.

### Restricted Maneuver Capability

Several encounter geometries were rerun to compare the performance of the logic when TCAS is in a climb-inhibited flight regime or is descend-inhibited due to its proximity with the ground. The testing showed TCAS performance to be significantly degraded in certain geometries, compared to its normal performance. These geometries are the obvious cases: where TCAS should climb but can not; or should descend but can not do so. The situations in which this occurs should be infrequent, since very few aircraft types are climb-inhibited, and only in rare conditions; and the descend-inhibit applies for only a very narrow band of altitude.

### Fault Tree Calculations

This section collects the various elements contributing to NMACs and calculates their total probability relative to the NMAC risk without TCAS. The process draws upon the original fault tree developed for the TCAS Safety Study, with certain changes to account for the more sophisticated analysis of logic performance. The original fault tree separated NMAC events into two categories termed "Unresolved" and "Induced." The present study examines TCAS logic performance in a more comprehensive manner and recognizes that some of the resulting NMACs cannot be meaningfully categorized as either Unresolved or Induced. Accordingly, this study drops the distinction between Unresolved and Induced NMACs for cases related to altimetry and logic performance.

Table ES-1 presents the results of the probability calculations of event chains for the Critical NMAC. The figures within the boxed areas represent these NMAC probabilities. They are totaled in the Summary section below to produce the final risk figure for the condition of all-RA's followed.

**Table ES-1. Fault Tree Calculation of Risk Ratio**

| | V 6.0 | V 6.04 |
|---|---|---|
| Unresolved NMAC related to non-logic factors: threat's lack of Mode C equipage and surveillance limitations | .0918 | .0918 |
| NMAC related to altimetry and logic performance | .0061 | .0173 |
| TOTAL | .0979 | .1091 |

These results show that logic-related NMACs are small compared to residual risks not related to the logic. Figure ES-3 illustrates the relative proportions. The change to v6.04 has little

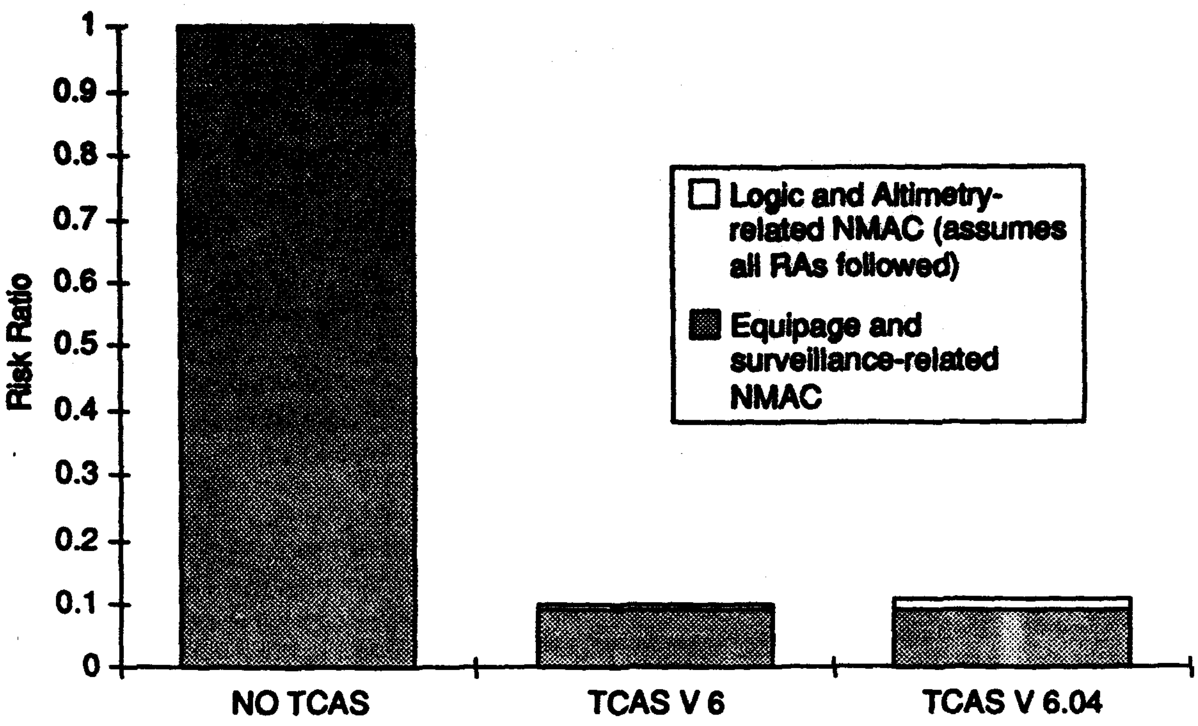



**Figure ES-3. Risk Ratio Components**

impact on the total risk. The contribution of non-logic related factors (Mode C equipage, surveillance, visual acquisition) dominates the altimetry and logic-related factors.

The result shown, based upon the site-average figures, for the increment of v6.04 over v6.0 is approximately one percent of the no-TCAS risk. This result is fairly consistent for all the sites studied: the largest site increment is 1.7 percent. Obviously, even a small degradation in protection would not be worthwhile were it not more than compensated by the benefits sought.

The lowest altitude layer is the greatest contributor to risk. The model may be pessimistic for this layer, where ATC exercises tight control. If this layer is excluded from the calculation, the risk ratio increment for v6.04 is 0.6 percent.

It must be emphasized that these calculations are based on the condition that all TCAS RAs are followed except those for which the pilot can recognize that to follow the RA would be unsafe. The next section makes important observations that modify these results for more realistic conditions related to the frequency of following RAs.

## EFFECTS OF NOT FOLLOWING RESOLUTION ADVISORIES

The simulation analysis described above applies to conditions where TCAS RAs always are followed, unless visually recognized as unsafe to do so. However, early operational experience using v6.0 of TCAS has shown the need for a better match with normal ATC operations, and that some significant fraction of TCAS RAs are not followed.

Figure ES-4 shows, as more RAs are followed, TCAS provides a successively increasing level of effectiveness, with former NMACs resolved, and a corresponding decrease in the remaining risk. Although the logic and altimetry-related NMACs increase, their frequency is considerably lower than the non-logic related unresolved NMACs.

Figure ES-5 compares the decrease in Risk for the two versions of logic when treating the fraction of RAs followed as a continuous variable. The scale is exaggerated for clarity in illustrating the following concept. At any constant value of RAs followed, v6.04 has higher risk; however, at the point where "X" percent of RAs are followed using the present (v6.0) logic, there is a corresponding point labeled "Y" percent where the v6.04 logic has identical Risk. Since the 6.04 logic is intended to eliminate a large fraction of the nuisance alerts, it is anticipated that TCAS advisories will be followed more frequently using v6.04. It would require only a small increase in this rate to achieve a lower risk in practice than is now being achieved using v6.0.

Figure ES-6 shows the actual plot of the site average Risk variation, drawn to scale. This figure uses as its endpoints the Risk Ratio result that is computed for the condition of all aircraft following their advisories. The two curves are extremely close together, reflecting the dominance of non-logic contributors to risk, principally non-transponder and non-Mode C equipped aircraft, and TCAS Mode C surveillance limitations. For these curves, even a one percent increase in the fraction of RAs followed would decrease the overall risk. A five percent increase in the percentage of RAs followed would decrease overall risk by about four percent.

### Coordinated Encounters

The following factors were considered for TCAS-TCAS coordinated encounters:

#### Surveillance Failure

Mode S surveillance is extremely reliable. The probability is of the order $10^{-3}$ or lower that either TCAS would fail to track the other, or would lose track before issuing an RA.

#### Coordination Link Failure

Approximately $10^{-13}$ TCAS-TCAS RAs would be expected to have both uncoordinated vertical sense and inadequate horizontal separation. This is smaller by many orders of magnitude than most other risks considered in this study.

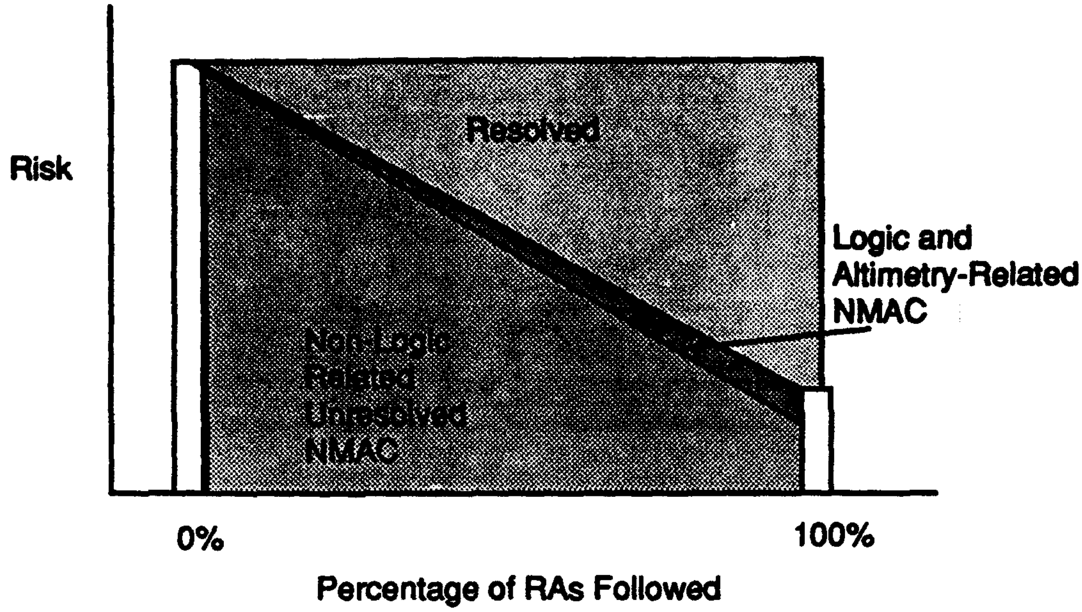



**Figure ES-4. NMAC Risk According to Fraction of RAs Followed**

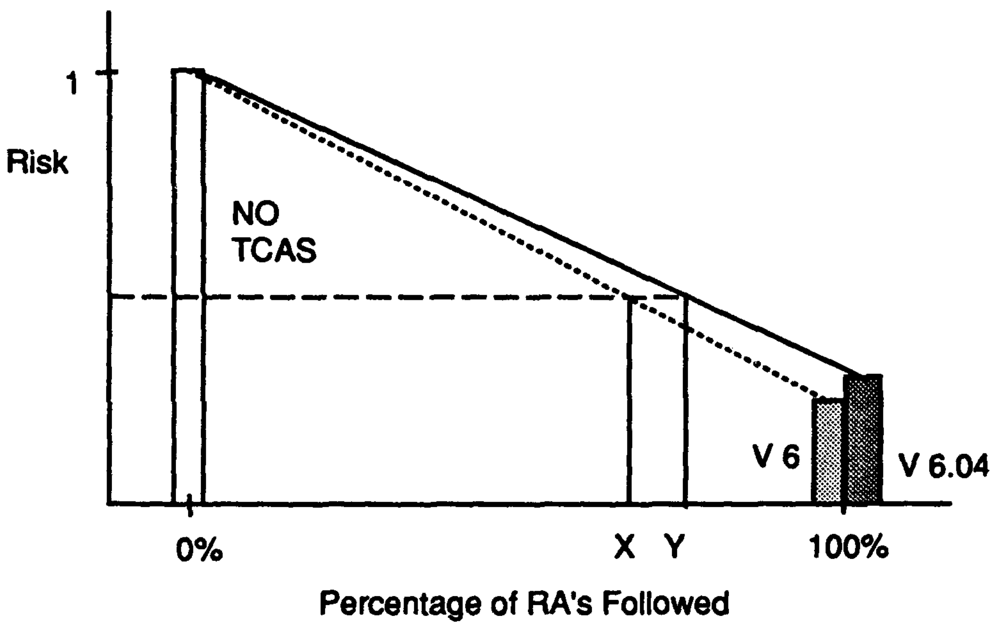



**Figure ES-5. Risk Comparison With Different Compliance for Alternate Logic Versions**

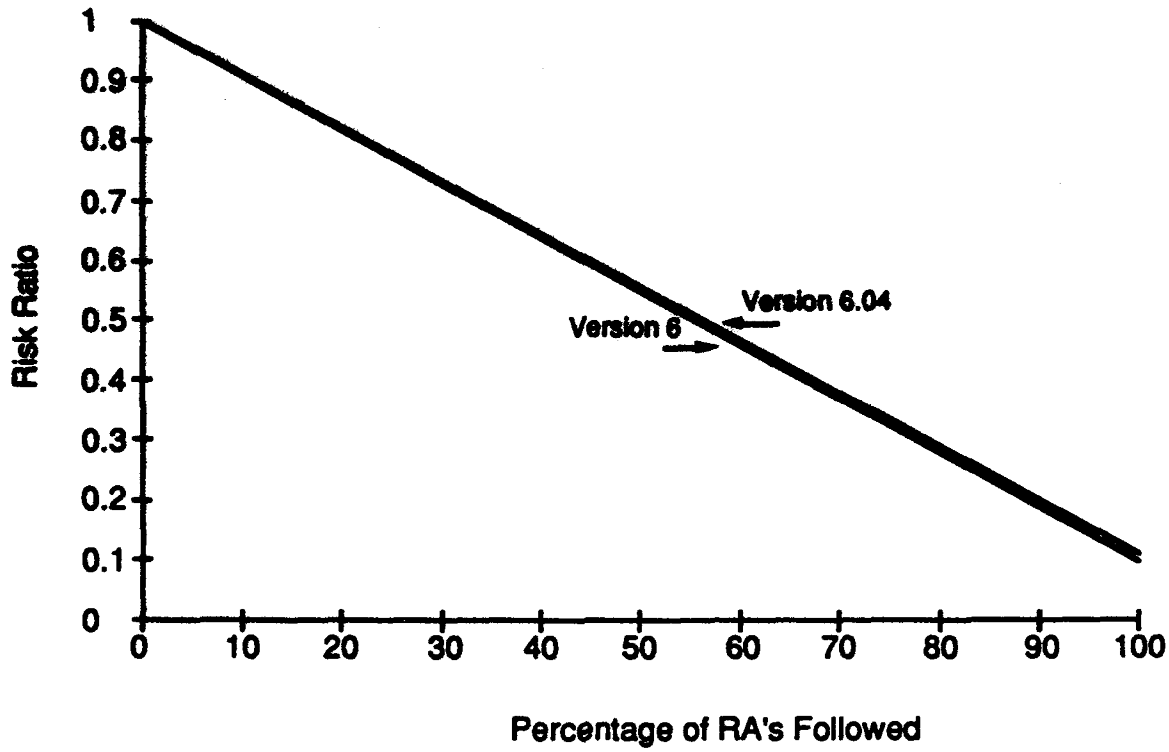



**Figure ES-6. Risk Variation According to Fraction of RAs Followed—Site Average Risk**

### Failure to Follow TCAS Advisory

The Federal Aviation Administration (FAA) Technical Center has performed extensive simulations of such encounters and have concluded that in virtually every encounter geometry, the two aircraft could have avoided an NMAC by following their respective TCAS advisories. This leads to the conclusion that a maneuver contrary to the direction advised by TCAS is likely to reduce the vertical separation.

This failure category has potentially greater significance than any other failure studied for the TCAS-TCAS encounters. Unfortunately, its magnitude is difficult to measure and especially difficult to predict. There is no evidence that the change of logic version would increase risk of this type. To the contrary, if v6.04 promotes increased pilot confidence in TCAS, as is intended, more of its advisories may be followed, with a safety benefit that is more-than-proportional.

**Unsafe Resolution Advisories**

**A coordinated TCAS-TCAS encounter should always produce a successful resolution if the advisories are followed. The FAA Technical Center has conducted extensive simulation of encounters to search for either routine or extreme conditions which could bring about an unsuccessful resolution. The only such geometry that was reported to fail involved an aircraft initially climbing at a rate of 5000 feet per minute (fpm) and encountering a level aircraft as it began to level off with a 1/3 g acceleration. Close encounters of this type are extremely infrequent in comparison to the routine encounters that contribute to the Risk Ratio component for unequipped intruders. Comparing these simulation results with those performed at The MITRE Corporation, their nearly perfect rate of successful resolution indicates that the logic-related component of Risk Ratio should be smaller by several orders of magnitude than the performance of the logic in encounters with unequipped threats.**

## CONCLUSIONS

This study has examined over 10,000 aircraft encounters at eight sites having different environments with respect to encounter geometries. Encounter modeling was conducted based upon 1889 RA-producing encounters. 780,000 Simulation runs using the complete logic have exercised a wide variety of geometry types for unequipped threats.

1. For the condition that all TCAS RAs are followed unless recognized as unsafe, logic v6.04 would produce a Risk Ratio only about one percent greater than for v6.0, on a theoretical site-average basis.

2. The greatest contribution to the v6.04 increment comes from the altitude layer below 2350 ft, where the lowest warning time is used. Since the ATC system is highly structured in that airspace, using the overall distributions of encounter classes and vertical rates may be unrealistic and give pessimistic results. Excluding the lowest layer, the Risk Ratio increment is about 0.6 percent.

3. The variation of this Risk Ratio increment among the sites studied was not great, despite very substantial differences in the encounter geometry proportions that were found. The greatest change in Risk Ratio for any of these sites was 1.7 percent. This gives confidence that studying other locations also would yield results very similar to the average figure. Furthermore, the increment due to the new logic version is of the same order as the normal site-to-site variations.

4. Recognizing that today pilots frequently do not follow RAs, often because of low confidence in TCAS, the achieved level of safety may be far from the ideal. If v6.04 raises pilot confidence to the point where even a few percent more RAs are followed, the achieved level of safety would improve, more than compensating for the reduction in warning time that eliminates many unnecessary RAs.

5. For coordinated TCAS-TCAS encounters, the logic, surveillance, and coordination functions are extremely safe and should not increase the Risk Ratio. The greatest hazard in this situation would be the failure to follow RAs. This may be alleviated if v6.04 brings about increased compliance.

6. The Risk Ratio component due to logic is only slightly degraded by imperfect surveillance quality. The relative performance of the two logic versions appears unchanged.

7. When TCAS is in a climb-inhibited flight regime, or is descend-inhibited due to its proximity with the ground, its performance is significantly restricted. Such situations should occur very infrequently relative to the rate of close encounters addressed in this study.

# SECTION 1

# INTRODUCTION

The Traffic Alert and Collision Avoidance System II (TCAS II) is becoming widely deployed among the U.S. Air Carrier fleet in accordance with Federal Aviation Administration (FAA) regulations. Smaller numbers of General Aviation (GA) and International users also are equipping with TCAS II (hereafter termed TCAS). The growing body of experience gained using TCAS in daily operations, however, has uncovered certain tendencies in which its Resolution Advisories (RAs) are often judged undesirable and potentially distracting. This operational experience evolved into new requirements for the TCAS logic which have been addressed by the proposed Version 6.04 (v6.04) [1]. The new logic is intended to better match TCAS with the Air Traffic Control (ATC) system.

This study assesses the safety of TCAS in domestic U.S. airspace. It provides specific comparisons between the previous (version 6.0 [v6.0]) logic and v6.04. Together with companion studies of operational characteristics [2], this Safety Study will give insight into the effects of using the 6.04 logic.

The 1983 Safety Study of TCAS II [3] defined the criterion of Risk Ratio as the metric of risk. That study also defined the critical Near Midair Collision (NMAC) as the root event, and created a Fault Tree, which enumerated combinations of credible events that could lead to an NMAC. The study used available data from various sources to evaluate most of these branches. Other elements, principally addressing factors such as crew actions, were evaluated parametrically.

In 1988, a Safety Study Update [4] was performed to evaluate significant changes in TCAS logic and to incorporate new data both for aircraft altimetry error and for encounter separation. The logic changes included new methods for selecting advisory sense in potential vertical crossing situations; for selecting advisory strength in converging situations which often developed into safely separated passages; for reversing advisory sense during an encounter; or for advising an increased escape maneuver during an encounter. This study also considered the effects of increased transponder equipage among the airspace population as a result of an FAA Proposed Rule.

## 1.1 PURPOSE OF STUDY

The primary purpose of the current Safety Study is to enable a comparison of the relative safety of TCAS v6.04 logic with the currently used v6.0. Version 6.04 contains some corrections which can only enhance safety, such as in correctly modeling the maneuvers considered during climb-inhibited or descend-inhibited conditions. However, the factor of greater concern is the decreased warning time afforded by TCAS v6.04, which results from

threshold reductions designed to eliminate the undesirable advisories. The most pertinent of these changes can be summarized as follows:

- Using lower sensitivity levels, with smaller protection volume, in certain low altitude regions

- Decreasing the warning time parameter ("TAU")

- Decreasing the vertical threshold for positive advisories (altitude limit "ALIM"), which represents the nominal vertical separation that TCAS attempts to achieve

- Further decreasing the warning time (TAU) in certain encounter geometries:
  - — For the level aircraft against a vertically converging threat
  - — For the aircraft with the lower vertical rate when a threat is converging with the same vertical sense

The purpose of these changes is to improve the match between TCAS and the ATC system. Some of the observed incompatibilities have been systematic, such as:

- Advisories at low altitudes causing unnecessary go-arounds

- Mixed instrument flight rules (IFR)-visual flight rules (VFR) traffic causing excessive, unnecessary advisories

- Advisories such as the "bump-up" disrupting ATC operations at higher altitudes

The approach to the changed logic is to selectively reduce thresholds, as described above, so that the greatest improvement in compatibility can be achieved with minimal change in the protection TCAS affords.

This study makes use of a significant quantity of data collected from the Automated Radar Terminal System (ARTS) in characterizing aircraft encounter geometries. These data represent an improvement over earlier studies with respect to the validity of the distribution of aircraft geometries in encounters. The study also employs computer modeling and simulation to evaluate the effects and complexities of TCAS logic over a wide variety of situations. While TCAS does not assure separation in every instance, the purpose of the selected approach is to account for many possible situations in their appropriate proportion.

This method of modeling the complete logic performance allows a more comprehensive examination of the causes and extent of circumstances for which the logic, in combination with altimetry error, fails to provide adequate vertical separation.

## 1.2 ORGANIZATION OF DOCUMENT

This report assumes some familiarity with TCAS and with the concepts of the Fault Tree and Risk Ratio from the previous TCAS Safety Studies. This section provides an overview. Section 2 describes the Methodology used in the study, including the encounter classification, the modeling of encounters, the simulation of close encounters, and the calculation of risk from simulation results. Section 3 provides the results of these same activities. Section 4 contains special analyses of failure to follow RAs, and of encounters against another TCAS-equipped aircraft. Section 5 contains the Conclusions of the study. Appendix A contains an analysis of altimetry error on safety, assuming nominal logic performance. Appendix B contains additional details of the simulation testing summarized in section 3. Appendix C presents statistics summarizing the results of the simulation testing for v6.04. Appendix D derives the equations governing the effects of two-aircraft altimetry errors.

# SECTION 2

# METHODOLOGY

## 2.1 OVERVIEW

This section describes the methods used to evaluate the relative performance of TCAS logic versions with respect to NMAC risk. The two principal concerns, as in the earlier studies, remain estimating TCAS' ability to resolve NMAC geometries, and estimating the probability that following a TCAS advisory would induce an NMAC that otherwise would not have occurred.

The principal innovative method underlying much of this analysis is the modeling and simulation of TCAS logic performance for close encounters. The pertinent characteristics of encounter geometries were extracted from radar data, enabling the definition of a set of encounter classes. These are then used in computer simulation to test large numbers of encounters in each class. With this method, the performance of TCAS and its safety implications for close encounters can be predicted with much greater confidence than by simply observing the more varied encounters that routinely occur. Close encounters are relatively rare, and the performance of TCAS logic in other encounters is not identical to its performance in the close ones. However, the larger set of encounters is useful for creating models of aircraft vertical profiles.

Since the study is projecting TCAS performance for close encounters, it would be ideal to collect a large database of them. However, their scarcity requires the pooling of data from encounters with greater separation. However, the database formed using v6.0 logic has 99 percent of its encounters that are only within 2 nautical miles (nmi). The approach used pools aircraft profile data (rates, accelerations) across all altitudes and sites. These are used to create models for simulating encounters with TCAS logic. On the other hand, the class distributions and vertical separation distributions, which appear to be more a function of ATC procedures and traffic mix, are examined separately by site. These distributions are used in post-processing the simulation results, making it more feasible to examine variations.

The process follows several steps, which are described in the indicated sections:

- Characterize the airspace using encounters found from ARTS data (section 2.3)
- Define models for simulating close encounters, using observed distributions of parameters (section 2.4)
- Simulate close encounters, primarily NMACs or those for which TCAS might induce an NMAC (section 2.4)

- From the distribution of perceived separation, analyze the effect of altimetry errors and calculate the probability of NMAC (section 2.5)

- Combine simulation results in proportions corresponding to observed encounter distributions (section 2.5)

- Apply fault tree factors and combine with non-logic limitations in resolving NMACs (section 2.6)

Figure 1 summarizes this process at a high level.

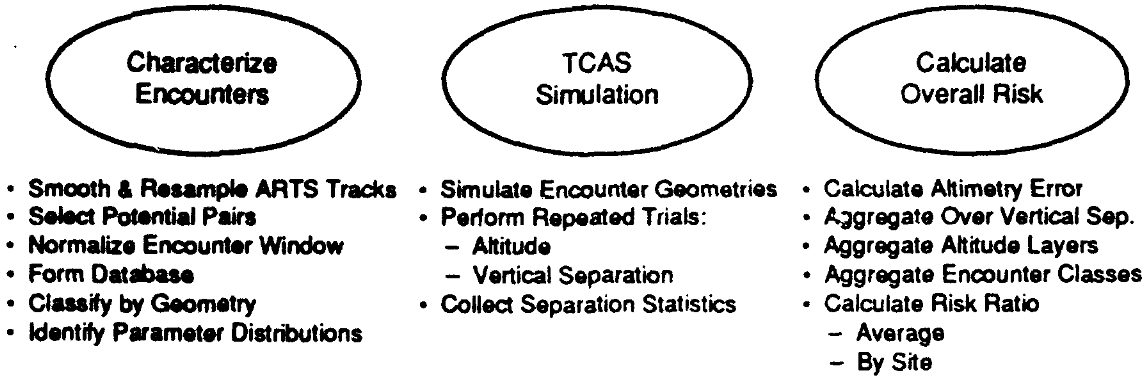



**Figure 1. Overview of Risk Calculation**

## 2.2 DATA SOURCES

In order to obtain a realistic model of the performance of TCAS, it is desirable to observe the behavior of aircraft and compile a database of pairwise encounters from the airspace of interest. Prior to this study, there existed only a relatively restricted database of European encounters. As part of the current effort, The MITRE Corporation has generated a large database of encounters from aircraft tracks recorded at a number of radar sites throughout the U.S. A brief description of this database is given here. Details are being published separately [5].

ARTS sites used in this study are at major airports and they cover the surrounding airspace out to a distance of approximately 45 nmi. The scan rate is typically 12 to 13 rpm. Using a secondary radar, the ARTS system also records the altitude (Mode-C) transmissions of those aircraft with Mode-C transponders. These altitudes are rounded off to the nearest 100 feet by the aircraft prior to transmission and they are scanned synchronously with the range/azimuth reports sensed by the primary radar. The result is a sequence of X, Y, Z, and T for nearly

every aircraft in the vicinity of the airport with updates every four to five seconds. From these raw data, other quantities of interest (e.g., speeds, accelerations, etc.) may be derived.

Eight sites were selected for this study: Burbank (BUR), Coast (CST) (in the Los Angeles Basin), Denver (DEN), Dallas-Fort Worth (DFW), New York (JFK), Minneapolis-Saint Paul (MSP), Seattle (SEA) and Saint Louis (STL).[1] These sites were chosen to represent moderate to heavy traffic under differing conditions of geography, traffic type, and ATC procedure.

ARTS data do not have the 1-Hertz update rate of TCAS tracks. Therefore, all of our recorded tracks were subjected to a spline-fitting procedure and resampled at 1 Hz. The resulting data are somewhat cleaner than are the target reports seen by TCAS. In discussing the simulation (section 3.4), this study addresses the effects of degraded surveillance.

The resampled tracks were considered pairwise using a coarse filtering procedure which follows the basic form of TCAS detection logic. This process forms candidate pairs for TCAS encounters; it selects six to eight times as many candidates as actually give RAs using v6.0 logic, and, therefore, should not miss any pairs (unless a track was missing or very short). Pairs for which both aircraft were identified as GA were excluded at this point, since these aircraft are unlikely to equip with TCAS II.

## 2.3 MODELING

The analysis outlined in section 2.1 for those aspects of TCAS safety addressed by this study requires a general scheme for describing encounters plus models for four quantities: |VMD| weights ($w_v$), layer weights ($w_l$), class weights ($w_c$), and altimetry error. Each of these is described below.

### 2.3.1 Classifying Encounters

The performance of TCAS depends, in part, on the geometry of the encounter and the maneuvering of aircraft during the encounter. These factors have been combined into the concept of "encounter class".

Since the performance is being compared of two versions of TCAS to each other as well as to "No TCAS", it is essential to have firm, unambiguous definitions for "encounter" and "encounter class". We begin by normalizing the time dimension with a standard encounter "window". This window is presented schematically in figure 2.

[1] Additional sites from the Los Angeles Basin were not used here, to avoid any risk of biasing the database towards one region.

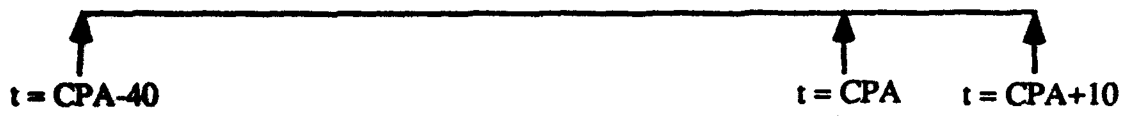



**Figure 2. Encounter Window**

The encounter window has its origin at the time of closest point of approach (CPA). From this point, it extends backwards 40 seconds and forward 10 seconds. This window contains the interval that affects the selection and execution of maneuvers to resolve the encounter. It also is unlikely that more than two vertical profile segments would be flown in this short a window. The database includes only encounters for which both aircraft were present throughout the entire window. In the descriptions which follow, the three times shown above (from left to right) are referred to as points A, B, and C. Whichever aircraft was the more "level" at point A is designated as Aircraft #1.

To determine a reasonable threshold for "level", a large number of plots of recorded encounters were examined. It was found that vertical speeds less than 400 fpm could not be reliably discerned given the quantization noise. Therefore, aircraft having $|\mathrm{ZDot}| < 400$ fpm are defined to be "level"; the rest are "transitioning". These definitions permit the classification of all pairwise encounters according to (a) the level/transitioning status of each aircraft before and after CPA and (b) the presence of an altitude crossing. Since "level" is defined to include any vertical speed less than 400 fpm, it is possible that there could be a crossing even when both aircraft are said to be level. The 20 encounter classes consistent with these definitions are listed in table 1.

In classes zero and ten, both aircraft are level before and after CPA. Given our procedure, we have no mechanism to force a crossing (or avoid a crossing) in these circumstances. Consequently, these two classes were combined. They will be referred to subsequently as class 10+0.

### 2.3.2 |VMD| Weights

TCAS logic operates in a symmetrical manner for TCAS above or below the threat. A more pertinent factor is the presence or absence of an altitude crossing during the encounter; this is covered by the definition of classes. Within a class, the absolute value of vertical separation at CPA, |VMD|, is used as the primary controlling variable. The simulation runs, described in section 2.4, cover geometries with vertical separations ranging from zero to 1000 feet.

This range is partitioned into ten equal bands. The results are combined as described in section 2.5, using the observed frequencies exhibited by the tracks for RA-producing encounters in the database. They vary from site to site and from class to class. For each site, this gives a vector of ten values for $w_v$ for each of the 19 classes. In addition, an unweighted average of these vectors, across sites, is used to compute a site-independent result.

**Table 1. Encounter Classes**

| Class | Aircraft #1 Before CPA | Aircraft #1 After CPA | Aircraft #2 Before CPA | Aircraft #2 After CPA | Crossing |
|---|---|---|---|---|---|
| 0 | L | L | L | L | ✓ |
| 1 | L | L | T | T | ✓ |
| 2 | L | L | L | T | ✓ |
| 3 | L | L | T | L | ✓ |
| 4 | T | T | T | T | ✓ |
| 5 | L | T | T | T | ✓ |
| 6 | T | T | T | L | ✓ |
| 7 | L | T | L | T | ✓ |
| 8 | L | T | T | L | ✓ |
| 9 | T | L | T | L | ✓ |
| 10 | L | L | L | L | |
| 11 | L | L | T | T | |
| 12 | L | L | L | T | |
| 13 | L | L | T | L | |
| 14 | T | T | T | T | |
| 15 | L | T | T | T | |
| 16 | T | T | T | L | |
| 17 | L | T | L | T | |
| 18 | L | T | T | L | |
| 19 | T | L | T | L | |

Notes: L = Level; T = Transitioning; Crossing refers to altitude crossing. In classes 2 and 12, either aircraft may be transitioning after CPA. In classes 6 and 16, either aircraft may be level after CPA.

Note that all classes are disjoint. This means that the total probability (of anything) over all classes is the sum of the respective probabilities for the individual classes.

These weights characterize the environment of close encounters prior to any action of TCAS. The character of this environment (since it was recorded prior to widespread TCAS equipage) is independent of TCAS logic. In this study, the RA-producing encounters of v6.0 form the database. Version 6.04 issues RAs for a subset of these encounters. Therefore, no close encounters critical to either logic should be missing from this database.

### 2.3.3 Layer Weights

Aircraft altitudes are addressed over six "layers" because TCAS logic parameters, and consequently its performance, are a function of altitude. The six layers are defined in table 2.

**Table 2. Altitude Layers**

| Layer | Lower Bound (ft) | Upper Bound (ft) | Weight ($w_l$) |
|---|---|---|---|
| 1 | 500/1000 | 2300 | 0.14 |
| 2 | 2301 | 5000 | 0.27 |
| 3 | 5001 | 10000 | 0.33 |
| 4 | 10001 | 20000 | 0.21 |
| 5 | 20001 | 30000 | 0.04 |
| 6 | 30001 | 35000 | 0.01 |

The layer weights, $w_l$, reflect historical proportions of reported NMACs. They are taken from the same database used in the altimetry analyses of the two previous Safety Studies. They are held constant and are assumed, for purposes of this study, to represent the altitude distribution of close encounters across all classes and sites.

The lower limit of layer 1 differs between v6.0 and v6.04. In this study, no provision is made for calculating the loss of protection in this narrow band of altitude, where TCAS by community consensus is taken out of its RA mode.

### 2.3.4 Class Weights

Class weights were determined simply by counting the RAs in each class in the database and dividing by the total number of RAs (see section 3.1). These weights were determined site by site. In addition, as an "average" number, a set of class weights averaged over all sites were calculated.

### 2.3.5 Parameter Distributions

Various parameters were extracted from the encounter database. The most significant for simulation modeling is the distribution of vertical rates. A distribution was developed for each profile (climb/descend, level, leveling, and level-to-transition). Relative range rates, vertical accelerations, and horizontal miss distances (HMDs) were also extracted for the creation of encounter models. All the distributions represent a pooling over all the sites and altitudes.

### 2.3.6 Altimetry Error

Aircraft altitudes, observed by either a ground radar or a TCAS receiver, are subject to errors caused by the altimeter system itself and by the quantization of the altimeter output (to the nearest 100 feet) in the Mode-C system. Previous TCAS safety studies examined the sources of altimetry error and estimated variances for the altitude error of a single aircraft and, hence, for the error in the vertical separation of two aircraft. By treating altimetry error as a random variable and assuming a form for the density function for the separation error ("overlap

density"), it was possible to estimate the contribution of altimetry error to NMAC probability given that the HMD was less than 500 feet and that the maneuvers made in response to TCAS RAs produced the nominal separation. A recalculation of this earlier method is performed in appendix A. The results of that calculation are consistent with this method.

In the present study, the effect of altimetry error is combined more explicitly with the performance of the logic. By incorporating full logic simulation, this avoids some of the simplifying assumptions that were made in the previous studies. The method, described in section 2.5.1 below, relies upon the observation that logic performance and altimetry errors are independent. This is true because altimetry errors are invisible to TCAS.

The net altimetry error, x, for one aircraft has a probability density which is approximately Laplacian (Double Exponential) with zero mean and a parameter, $\sigma$, that is a function of true altitude and the presence of "corrections" carried out by an Air Data Computer[2] (equation 1).

$$f(x) = (2\,\sigma)^{-1} \exp\left(\frac{-|x|}{\sigma}\right) \qquad 1.$$

Empirical values for sigma have been reported, for several altitude layers, for "corrected" and "uncorrected" altimetry [3], [4], [6], [7]. The values used here (in feet) are presented in table 3. Subsequent calculations are based upon the TCAS aircraft having corrected altimetry and the other aircraft having uncorrected altimetry 60 percent of the time. This partition reflects the observation, in the database, that 60 percent of the intruders were GA aircraft [5].

**Table 3. Altimetry Error Parameters**

| Layer | Altitude (ft.) | Sigma | |
|---|---|---|---|
| | | Corrected Altimetry | Uncorrected Altimetry |
| 1 | 2300 | 46 | 67 |
| 2 | 5000 | 48 | 67 |
| 3 | 10000 | 52 | 75 |
| 4 | 20000 | 65 | 92 |
| 5 | 30000 | 78 | 105 |
| 6 | 35000 | 86 | 105 |

2 Air Data Computers are present on carrier aircraft but not, usually, on lower performance GA aircraft.

## 2.4 TCAS ENCOUNTER SIMULATIONS

### 2.4.1 Simulation Description

The encounter simulator [8] performs repeated simulations of specified encounter geometries with numerous parameters being varied on successive runs to give a rich sampling of the multi-parameter sample space. Every sample encounter is run three times: without TCAS, and with each aircraft in turn carrying and responding to its TCAS, with the other aircraft unequipped. The same encounters are rerun for both the v6.0 and v6.04 logic. The simulated TCAS aircraft responds to any TCAS RAs generated according to a model which also provides a range of statistical variation. Separations are compared both with and without TCAS to enable the tabulation of encounter separation statistics.

The simulation uses an input file that specifies each encounter geometry to be run. Either a single, specific case can be run, or a series of repetitions can provide random sampling of many parameters. The present study models each encounter class as a nominal case, and uses parameter distributions to specify the variations in aircraft vertical rates, the time and magnitude of accelerations, horizontal speed, and miss distance. In this study, altimetry error is not simulated; instead, its effect is calculated on a statistical basis.

The simulation models the relative motion of the two aircraft in the encounter at one-second intervals from 50 seconds before until 40 seconds after their closest point of approach. When one aircraft is modeled as TCAS-equipped, the simulation calls the TCAS logic and passes it nearly all of the inputs that would be made available to that logic in an actual installation. These inputs include own aircraft's barometric altitude and radar altitude, the threat's reported range and quantized altitude report, and own aircraft's climb-inhibit status. The logic performs its normal functions, including the selection of sensitivity level and the associated logic parameters, threat detection, advisory selection, and display. This simulation does not include the advisory coordination function; accordingly, encounters against TCAS-equipped threats are not simulated here, but are described separately in section 4.2.

The simulation gives the option of testing imperfect surveillance by selecting a reply probability for the threat. When this probability is set to less than unity, each surveillance reply is independently determined to be received or be missed.

The path of the TCAS-equipped aircraft is modified to reflect the pilot's response to TCAS advisories. For this study, the simulated delay is varied uniformly over 4-6 seconds for an initial advisory, and with less delay in responding to any later advisory. The accelerations used are nominally 0.25 g in responding to an RA and for returning to the nominal flight path at the end of an advisory. The nominal value 0.33 g is used for "reversal" or "increase rate" RAs. Other parameter variations in the system account for transponder noise and delay, surveillance range errors in bias and jitter, and noisy radar altimeter data.

### 2.4.2 Encounter Model Creation

These simulations require modeling that thoroughly exercises the logic with a variety of parameter variations for each encounter geometry. Each aircraft uses independently selected values of speed, vertical rate, and maneuver time, where appropriate. Every value is drawn randomly from the distribution appropriate to its profile.

This study is specifically intended to measure the Risk Ratio. Therefore, only encounters with HMD less than 500 ft are relevant. The model forms geometries with HMD varying over 0 to 500 ft at the closest point of approach.

Encounter simulations are run for one Class (i.e., geometry) at a time. Each set of runs duplicates the geometry for ten "bands" of vertical separation for each Class. These bands represent the geometry's nominal vertical separation (VMD) without TCAS (figure 3). To saturate each band, 500 encounters are run in each, with the vertical separation randomly drawn from a uniform distribution. The first band uses vertical separations at CPA from 0 to 100 ft; the second band from 100 to 200 ft; up to the tenth band from 900 to 1000 ft.

These encounter Class simulations are repeated for each of the six altitude layers (table 3), for which TCAS uses different logic parameters.

The simulator collects the vertical separation at CPA for each repetition, both with and without TCAS (see figure 4). These results are combined as described in the next section.

## 2.5 CALCULATING RISK FROM SIMULATION RESULTS

### 2.5.1 Combining Effects of Logic Performance and Altimetry

The simulation output (measuring logic performance) is collected in the form of a histogram of |VMD| over a range of zero to 1,500 feet (150 bands) for each of the three cases of interest (viz., No TCAS, v6.0 and v6.04). The total NMAC probability for the class is the sum of the (disjoint) VMD band probabilities. Taking the midpoint of each band as the perceived separation, S, the overlap density and the relative frequency in the band is used to estimate the probability that the true (unobserved) vertical separation was $\leq 100$ feet given that S was observed (see figure 5). The overlap density is the convolution of the respective error densities for the two aircraft.

The total altimetry error is the sum of two random errors. Since the two aircraft errors are independent, the probability density of their sum equals the convolution of the random variables. In this case, we require the convolution of two Laplacian density functions. There are two cases to consider depending upon whether the parameters are equal or unequal. The convolution in the former case has been reported previously [9]. It is given in equation 2. In the following equations, z is the algebraic sum of the two altimetry errors.

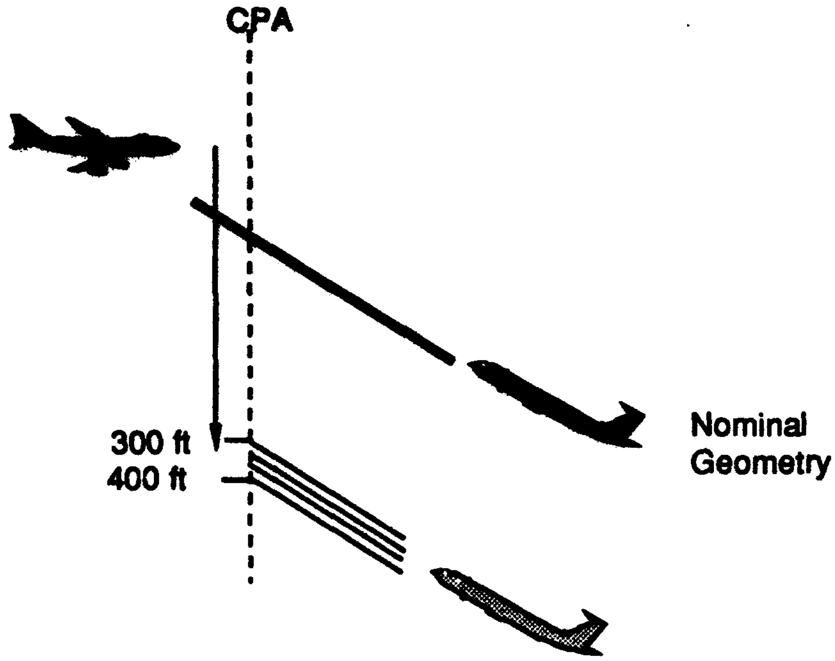



**Figure 3. Encounter Geometry Repeated Over Several "Bands" of Vertical Separation**

*Each geometry class is rerun many times to measure performance at each narrow band of vertical separation at CPA. In this example, the band from 300 to 400 ft separation is shown. Every repetition has further variations, such as vertical rates, accelerations, and maneuver times. Every case is repeated, with each aircraft in turn carrying TCAS.*

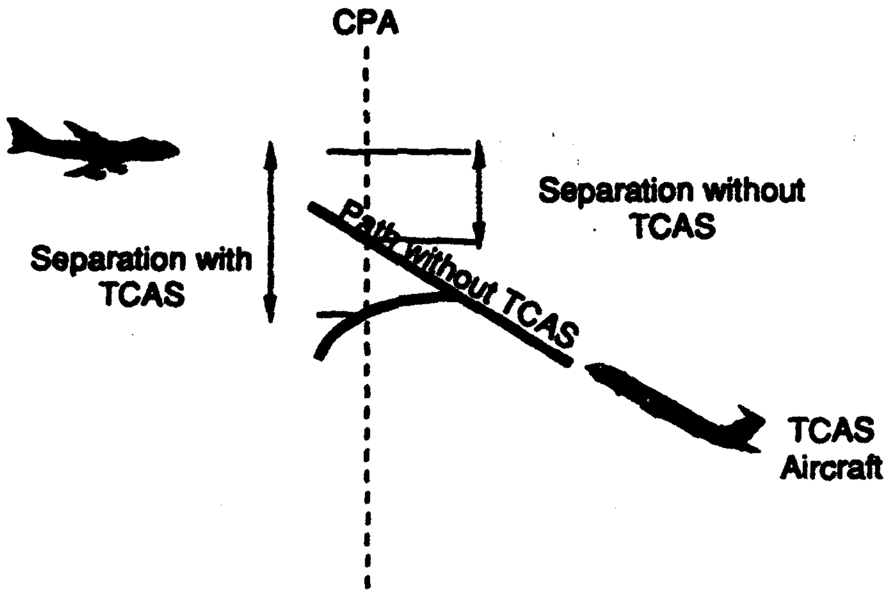



**Figure 4. Perceived Separation Resulting from TCAS RA**

*The simulation runs vary parameters such as pilot response times. For each band that is run, statistics are collected comparing the vertical separations from responding to TCAS RAs to those without TCAS. A single encounter is illustrated. These separations only represent those perceived from Mode C reports, as they do not include any effects of altimetry errors.*

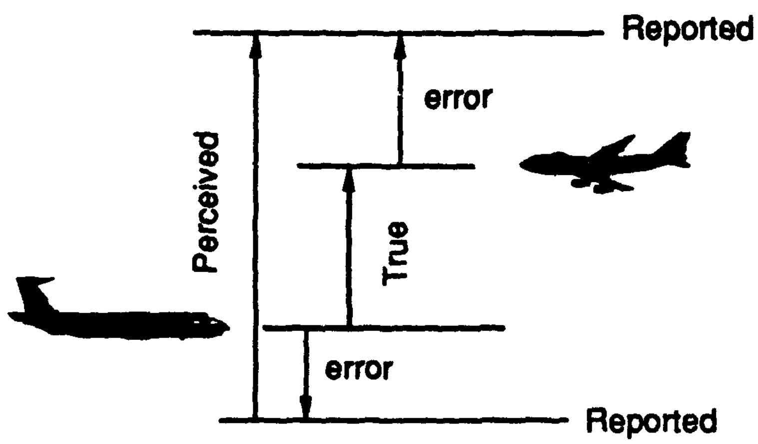



**Figure 5. True Versus Perceived Vertical Separation Due to Altimetry Error**

$$C(z) = (4\,\sigma^2)^{-1}\,(|z| + \sigma)\,\exp(\frac{-|z|}{\sigma}) \qquad 2.$$

The convolution for the case of unequal parameters is given in equation 3 (see appendix D).

$$C(z) = \frac{\sigma_1 \exp(\frac{-|z|}{\sigma_1}) - \sigma_2 \exp(\frac{-|z|}{\sigma_2})}{2\,(\sigma_1^2 - \sigma_2^2)} \qquad 3.$$

The probability that the true vertical separation of the aircraft is ≤ 100 feet (an NMAC) can be computed using equation 4. Closed-form solutions for equation 4 have been obtained and verified by

$$\mathrm{Prob}(|sep| \le 100) = \int_{S-100}^{S+100} C(z)\,dz \qquad 4.$$

comparison with the results of Monte Carlo simulations (see appendix D).

An example of the effect of altimetry error on NMAC probability is illustrated in figure 6. This figure shows the probability that true |VMD| is ≤ 100 feet as a function of perceived |VMD|. The example is for altitude layer 3 with $\sigma_1 = 52$ feet and $\sigma_2 = 75$ feet.

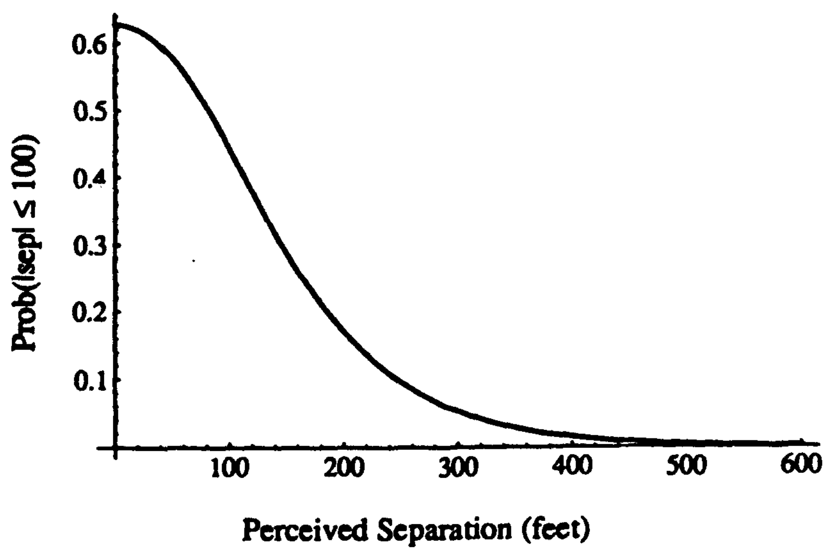



**Figure 6. An Example of the Effect of Altimetry Error on NMAC Probability**

Figure 7 illustrates the process in terms of vertical separation statistics for the 500 encounters run in a single band. Part (a) shows that the cases run in this band all would have a given |VMD| (in this case 300 to 400 ft) without TCAS. Part (b) shows the distribution of apparent separation after responding to TCAS RAs. This distribution is spread around a mode of about 600 ft. After applying the altimetry error distribution described above, the true separation distribution of this output is reflected in part (d). This shows that the distribution of true |VMD| is spread over a wider range than was the perceived distribution of (b). This output should be compared with that in part (c), which shows the distribution of altimetry errors applied to the No-TCAS distribution (a). Then for the distributions (c) and (d), the probabilities of |VMD| < 100 ft are calculated to produce the respective NMAC components. Using TCAS has clearly decreased the risk in this case (one of the simulation runs performed in this study).

### 2.5.2 Combining Results for Risk Ratio

This section describes the process used to collect the results of the many simulated encounters and develop an overall Risk Ratio statistic. The goal is to compare the vertical separations provided by the two logic versions, with the many encounter geometries put in proper context (i.e., more common cases weighted more heavily).
The Risk Ratio is defined by:

$$\text{Risk Ratio} = \frac{\text{prob [NMAC with TCAS]}}{\text{prob [NMAC without TCAS]}} \qquad 5$$

Both numerator and denominator can be expanded using conditional probabilities in the following form to account for encounter classes:

$$\text{prob [NMAC]} = \sum_{\text{classes}} \text{prob [NMAC | class]} \cdot \text{prob [class]} \qquad 6.$$

The following describes the procedure for determining each term prob [NMAC | class]. The vertical separation at CPA is measured for each simulation run, both with and without TCAS. The statistics are tabulated for the batch of 500 encounters in each band of VMD, designated below using the subscript "v", and run for altitude layer L. For this distribution, the altimetry error model is applied to calculate the probability that true separation is within the NMAC region. This is done twice: for threats with good altimetry and for threats with low quality altimetry. TCAS is always assumed to carry high quality altimetry.

The results of the two altimetry calculations are combined according to the ratio of RA-producing encounters seen in the ARTS data: 60 percent GA (low-quality altimetry assumed) and 40 percent high-quality altimetry. Then these results are combined for the VMD bands according to the frequencies observed at each site's data for this class of encounters.

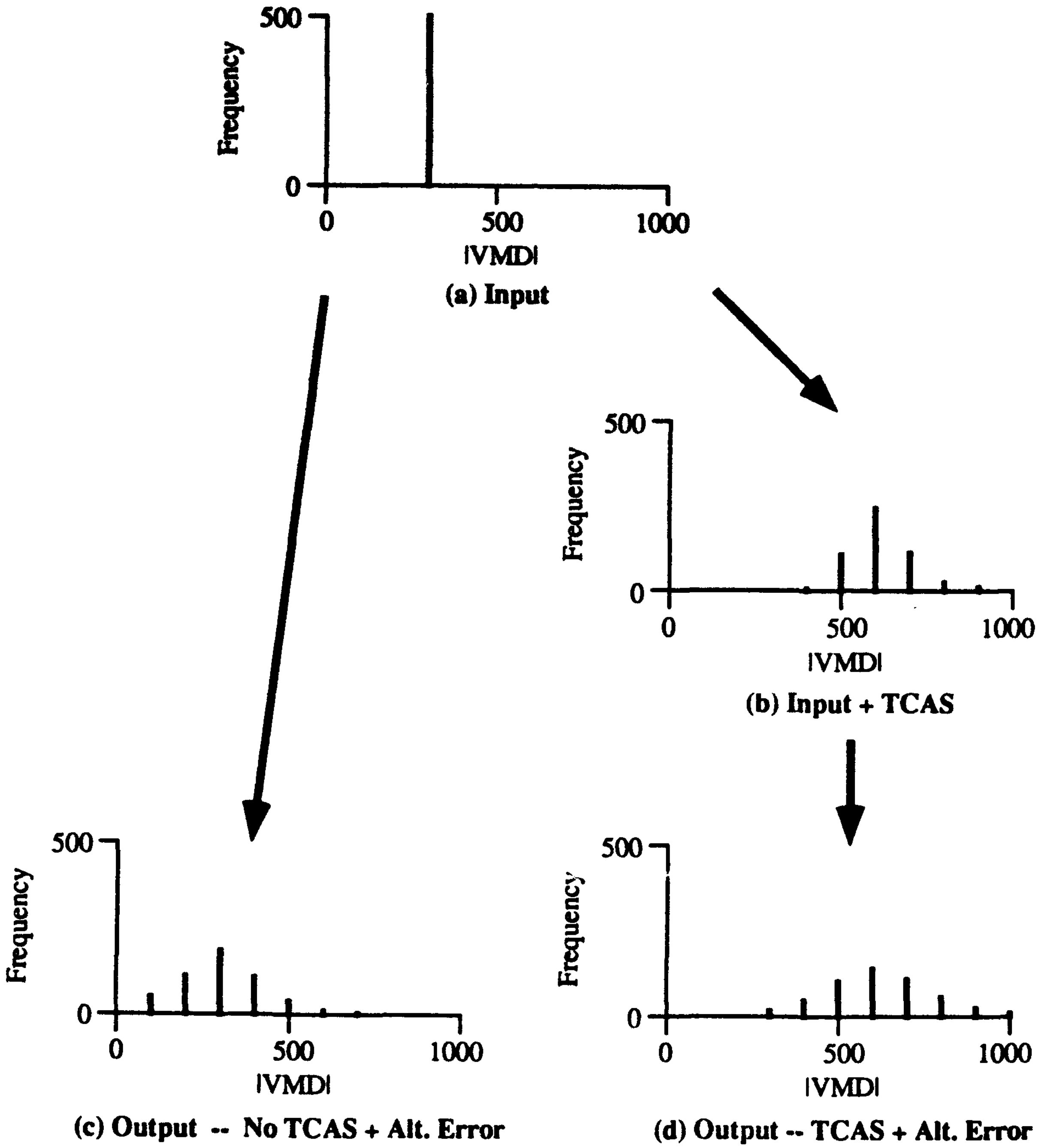



**Figure 7. Effect of TCAS and Altimetry Error on |VMD|**

This set of calculations can be expressed mathematically using the following notation with subscripts identifying various probabilities:

$P_{TvLa}$ = Probability of NMAC using TCAS version T, in VMD-band v, at alt. layer L, for altimetry quality = GB or GG (Good vs. Bad or Good vs. Good).

To calculate the result for an encounter class for each altitude layer, the first step combines the results of the two calculations for altimetry quality, again using conditional probabilities:

$$P_L = \sum_{v=1}^{10} [P_{vLGB}(0.6) + P_{vLGG}(0.4)]\, w_v \qquad 7.$$

where the TCAS version (T) has been omitted from the equation for brevity. The $P_{vLGB}$ and $P_{vLGG}$ terms are the risk calculations described above which apply altimetry error to the perceived separations. The $w_v$ values represent the observed VMD frequencies from the ARTS data (i.e., without TCAS), tabulated for each encounter Class (C). Figure 8 illustrates the process of combining these v-weights.

The next step is to aggregate the results of the simulation runs over the six altitude layers for each Class (C), giving the prob [NMAC | class] from above:

$$P_C = \sum_{L=1}^{6} P_L w_L \qquad 8.$$

This process is illustrated in figure 9. The $w_L$ values are NMAC frequencies for each altitude layer. These values are altitudes taken from the historical reports of NMACs. This was the same data used for the altimetry studies in the two previous Safety Studies.

$$(\text{Note: } \sum_L w_L = 1)$$

Next, calculate and aggregate these same results over all encounter Classes at Site S:

$$P_S = \frac{\sum_C P_C n_{CS}}{\sum_C n_{CS}} \qquad 9.$$

where the values $n_{CS}$ are the actual counts of RAs in each Class C at site S. These counts represent estimates for the prob [class] terms. This $P_S$ is the desired prob [NMAC] result.

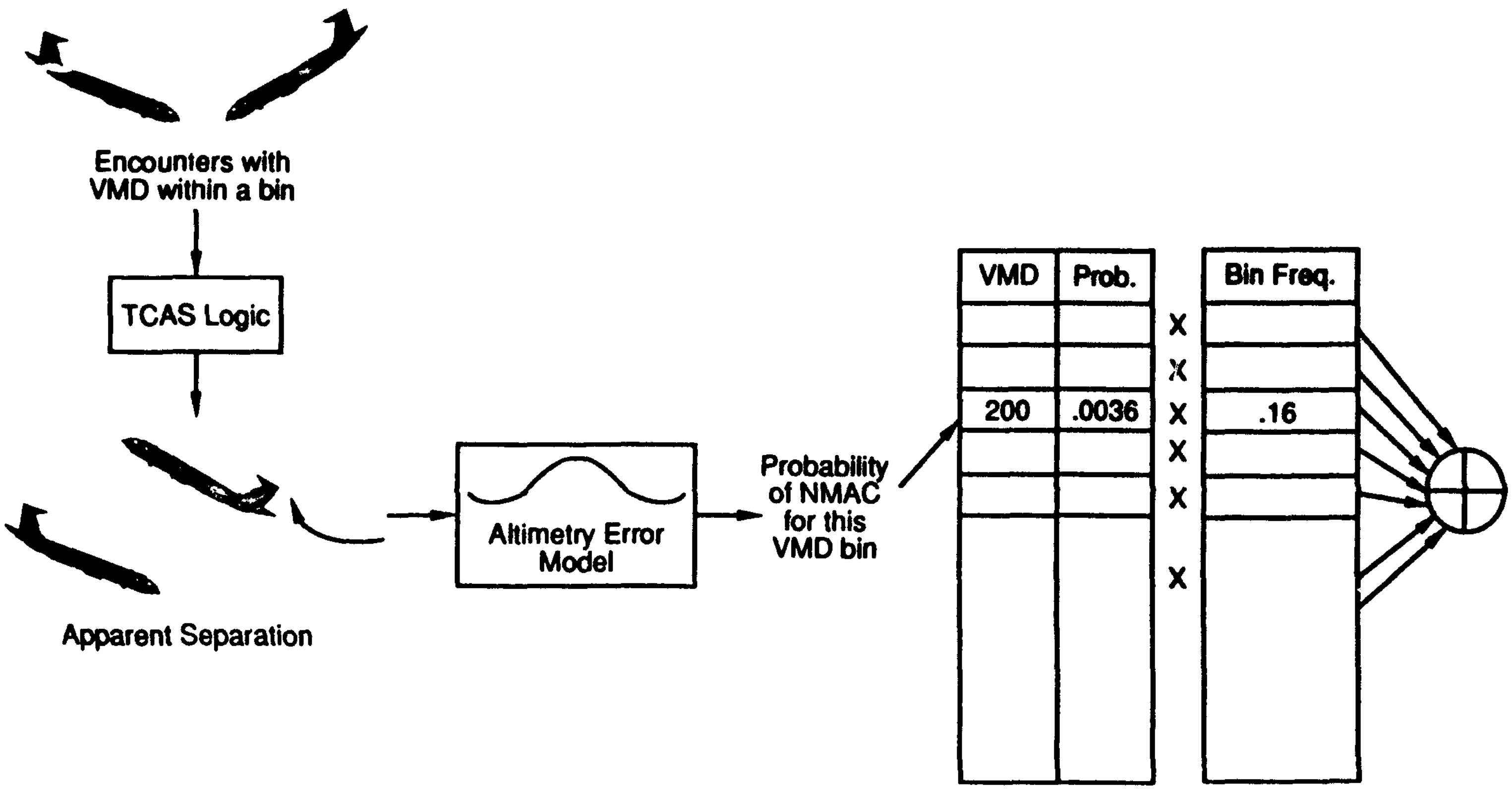



Figure 8. Weighting Results for Bins of Vertical Miss Distance

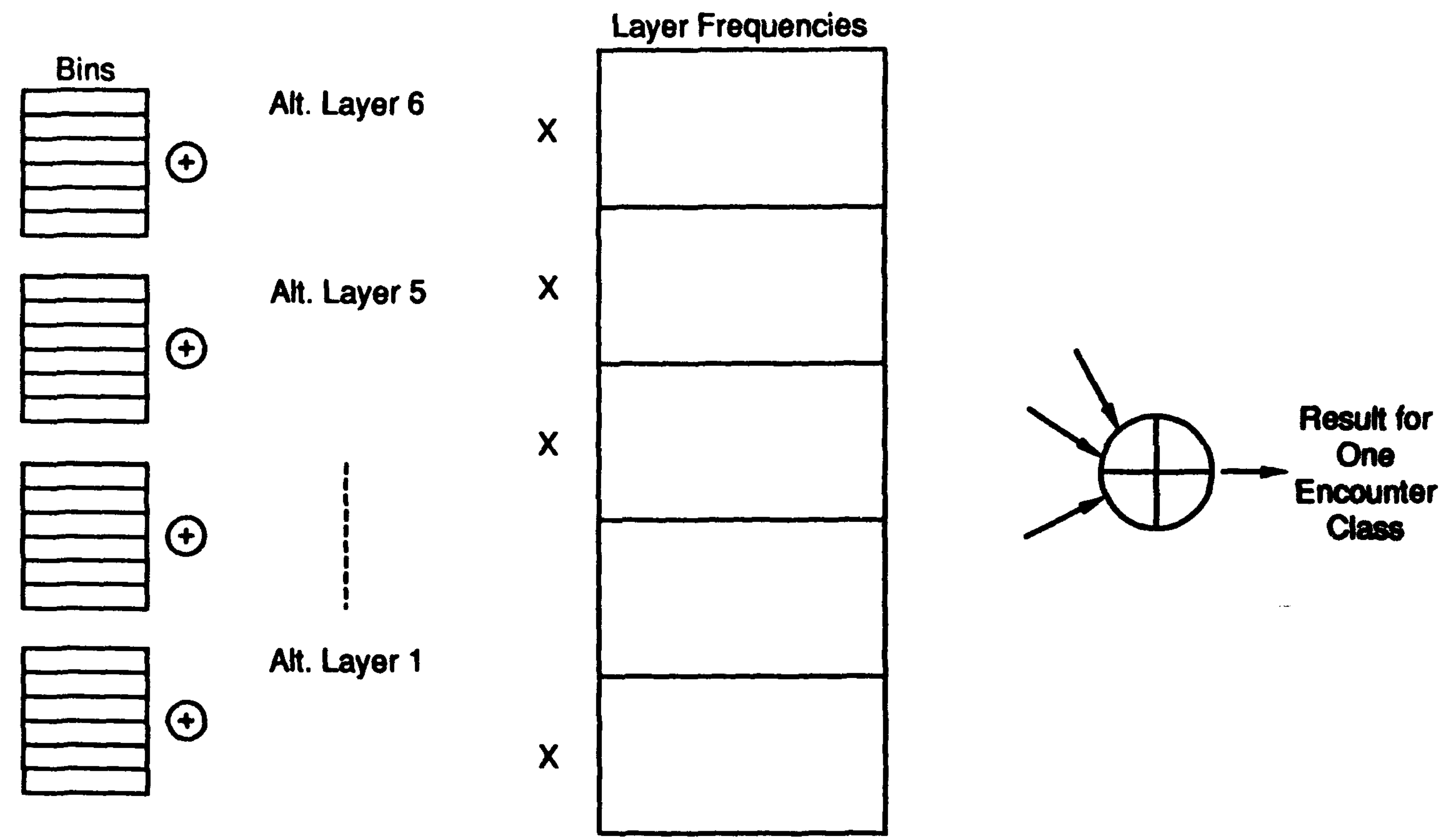



**Figure 9. Combining Results for Altitude Layers**

An investigation was performed to consider adjusting the prob [class] terms to account for potential changes in encounter frequencies for the small range of HMD represented by this calculation. The actual counts of RAs covered a wider range of HMD. A contingency table was formed which compared the class counts at small HMD with the class counts of all RAs. A Chi-square test determined that the hypothesis that class frequency does not depend on HMD could not be rejected. Therefore, having determined that this distribution will not bias the class weights, no adjustment was made in the class counts.

Finally, the NMAC probabilities are used to calculate Risk Ratios for the various TCAS equipages. Denoting the logic versions by the subscripts 6 and 6.04, and no-TCAS by subscript 0, the following form the Risk Ratios for each site (S):

$$RR_6 = \frac{P_{S\,6}}{P_{S\,0}} \quad \text{and} \quad RR_{6.04} = \frac{P_{S\,6.04}}{P_{S\,0}} \qquad 10.$$

## 2.6 FAULT TREE CALCULATIONS

As in the earlier Safety Studies, the calculation of NMAC related to logic performance and altimetry is conditioned upon many factors. The Fault Tree, shown in figure 10, provides the structure to evaluate the relevant factors and their conditional probabilities. In section 3.5, the calculations are performed using the same method as in the earlier studies, accumulating the joint or conditional probabilities of event chains which would lead to a NMAC.

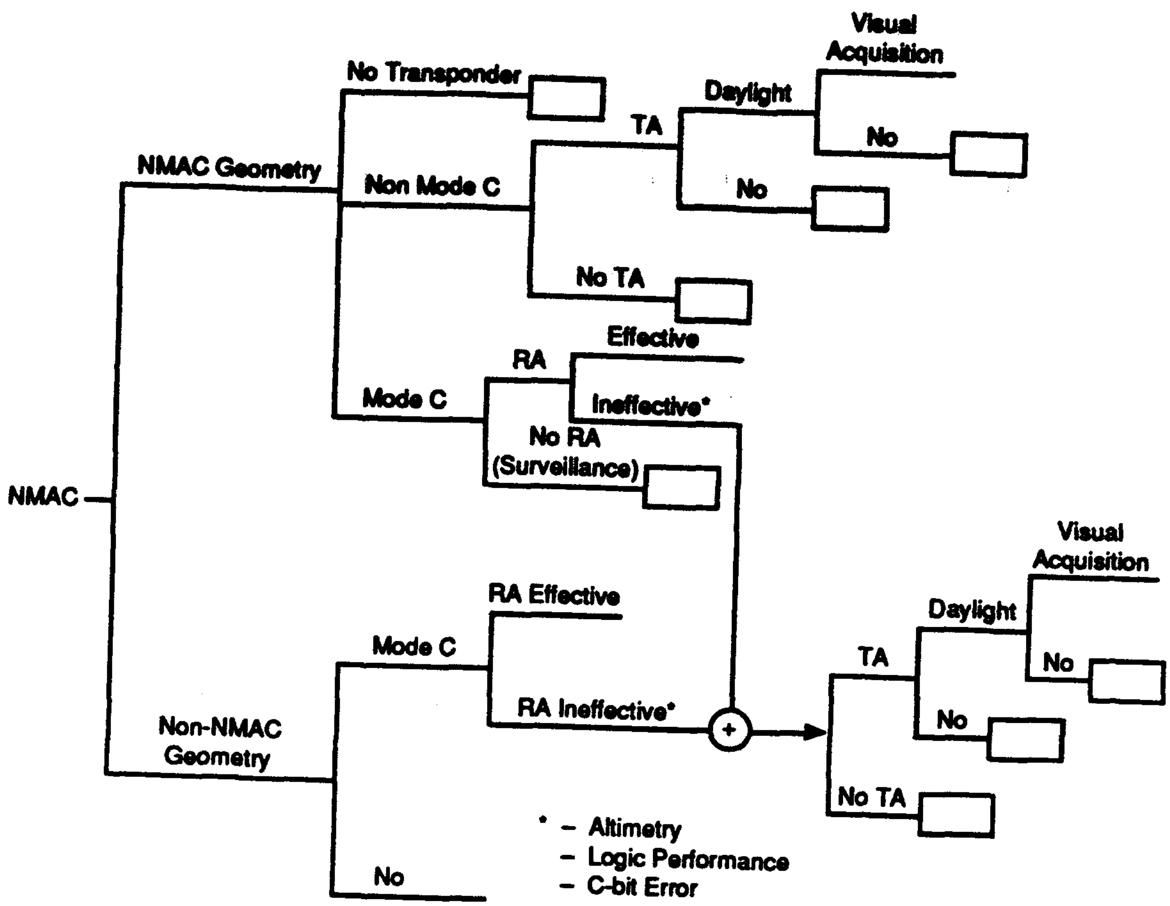



Figure 10. TCAS Fault Tree

## SECTION 3

## RESULTS

This section summarizes the findings in categorizing encounters and extracting their relevant parameters for modeling; the results of simulation runs of the various encounter classes; and the results of the process of calculating the Risk Ratio for unequipped threats, assuming the TCAS RAs are followed. The Risk Ratio results are presented for individual sites and as the average of the sites.

### 3.1 OBSERVED ENCOUNTERS

The distribution of encounters observed and the statistics derived from them are described below.

#### 3.1.1 Encounter Class Distribution

The class distributions of encounters and RAs, summed over all eight sites, are presented in table 4. "All Encounters" represent pairs which pass the coarse filtering criteria. The "RAs" are those encounters which would have generated a RA using the v6.0 logic. The RA columns of this table were further divided according to site, in order to compute the class weights, $n_{cs}$. There is little difference in the class distribution of "All Encounters" versus the distribution of "RAs." This lends validity to the assumption that this class distribution can characterize the encounter environment, and is not sensitive to the details of the logic version that was used to generate the "RAs" set.

Table 5 shows the site-by-site comparison of RA classes. Considerable differences are seen in the environments represented by this collection of sites. For example, Leveloff encounters (Class 13) are more frequently observed at Dallas than anywhere else. Level encounters (Class 10+0) predominate in New York and Minneapolis-St Paul. Pairs of descending aircraft (Classes 4 and 14) are much more common in St. Louis and Denver. Altitude crossings (Classes 1 through 9) are moderately frequent at Burbank, where more GA mix with air carriers than at these other sites. The differences of this environmental mix also can be seen from figure 11, which compares the proportions of some of the larger classes.

### 3.2 SIMULATION RESULTS

The basic simulation runs performed for this study (exclusive of degraded surveillance or climb- or descend-inhibited cases) totaled 780,000 encounters for each version of the logic. These runs included all classes with up to one maneuver, which represented 98 percent (1849 of 1889) of the RAs in the database.

**Table 4. Encounter-Class Distributions**

| Class | All Encounters Frequency | All Encounters Percent | RAs Frequency | RAs Percent |
|---|---|---|---|---|
| 1 | 291 | 2.7 | 129 | 6.8 |
| 2 | 26 | 0.2 | 7 | 0.4 |
| 3 | 43 | 0.4 | 21 | 1.1 |
| 4 | 189 | 1.8 | 90 | 4.8 |
| 5 | 48 | 0.4 | 23 | 1.2 |
| 6 | 37 | 0.3 | 17 | 0.9 |
| 7 | 4 | 0.04 | 2 | 0.1 |
| 8 | 6 | 0.1 | 5 | 0.3 |
| 9 | 2 | 0.02 | 1 | 0.1 |
| 10+0 | 3512 | 32.8 | 659 | 34.9 |
| 11 | 3069 | 28.7 | 331 | 17.5 |
| 12 | 753 | 7.0 | 131 | 6.9 |
| 13 | 941 | 8.8 | 158 | 8.4 |
| 14 | 964 | 9.0 | 213 | 11.3 |
| 15 | 324 | 3.0 | 37 | 2.0 |
| 16 | 324 | 3.0 | 33 | 1.7 |
| 17 | 45 | 0.4 | 7 | 0.4 |
| 18 | 74 | 0.7 | 13 | 0.7 |
| 19 | 46 | 0.4 | 12 | 0.6 |
| Total —> | 10698 | 100 | 1889 | 100 |

**Table 5. Percentage Encounter Classes by Site (RAs Only)**

| Class | BUR | CST | DEN | DFW | JFK | MSP | SEA | STL |
|---|---|---|---|---|---|---|---|---|
| 1 | 10.9 | 9.6 | 3.5 | 3.4 | 2.0 | 4.3 | 5.3 | 2.8 |
| 2 | 0.0 | 0.4 | 1.1 | 0.0 | 0.0 | 1.0 | 0.0 | 0.0 |
| 3 | 0.7 | 1.8 | 1.1 | 0.0 | 1.0 | 1.0 | 1.5 | 0.9 |
| 4 | 5.0 | 1.8 | 11.3 | 2.7 | 0.0 | 1.9 | 5.3 | 12.0 |
| 5 | 1.4 | 1.8 | 1.8 | 0.0 | 1.0 | 1.0 | 0.0 | 0.0 |
| 6 | 1.7 | 1.0 | 1.1 | 0.0 | 0.0 | 0.0 | 0.8 | 0.9 |
| 7 | 0.0 | 0.0 | 0.4 | 0.7 | 0.0 | 0.0 | 0.0 | 0.0 |
| 8 | 0.2 | 0.2 | 0.7 | 0.0 | 0.0 | 0.5 | 0.0 | 0.0 |
| 9 | 0.2 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 10+0 | 22.6 | 44.1 | 24.4 | 25.2 | 65.3 | 57.1 | 37.6 | 7.4 |
| 11 | 28.0 | 14.9 | 9.2 | 22.4 | 15.3 | 7.6 | 28.6 | 11.1 |
| 12 | 6.9 | 8.2 | 7.1 | 6.1 | 6.1 | 8.6 | 5.3 | 1.9 |
| 13 | 8.1 | 8.6 | 3.5 | 29.9 | 6.1 | 5.2 | 4.5 | 4.6 |
| 14 | 5.5 | 3.3 | 30.1 | 4.1 | 1.0 | 8.1 | 6.8 | 51.9 |
| 15 | 3.8 | 1.6 | 0.7 | 4.1 | 1.0 | 1.4 | 0.8 | 0.0 |
| 16 | 2.4 | 0.8 | 2.5 | 0.7 | 1.0 | 1.0 | 2.3 | 4.6 |
| 17 | 0.7 | 0.4 | 0.4 | 0.0 | 0.0 | 0.5 | 0.0 | 0.0 |
| 18 | 1.2 | 0.6 | 0.7 | 0.7 | 0.0 | 0.0 | 0.8 | 0.9 |
| 19 | 0.7 | 0.8 | 0.4 | 0.0 | 0.0 | 1.0 | 0.8 | 0.9 |
| # RAs | 421 | 490 | 282 | 147 | 98 | 210 | 133 | 108 |

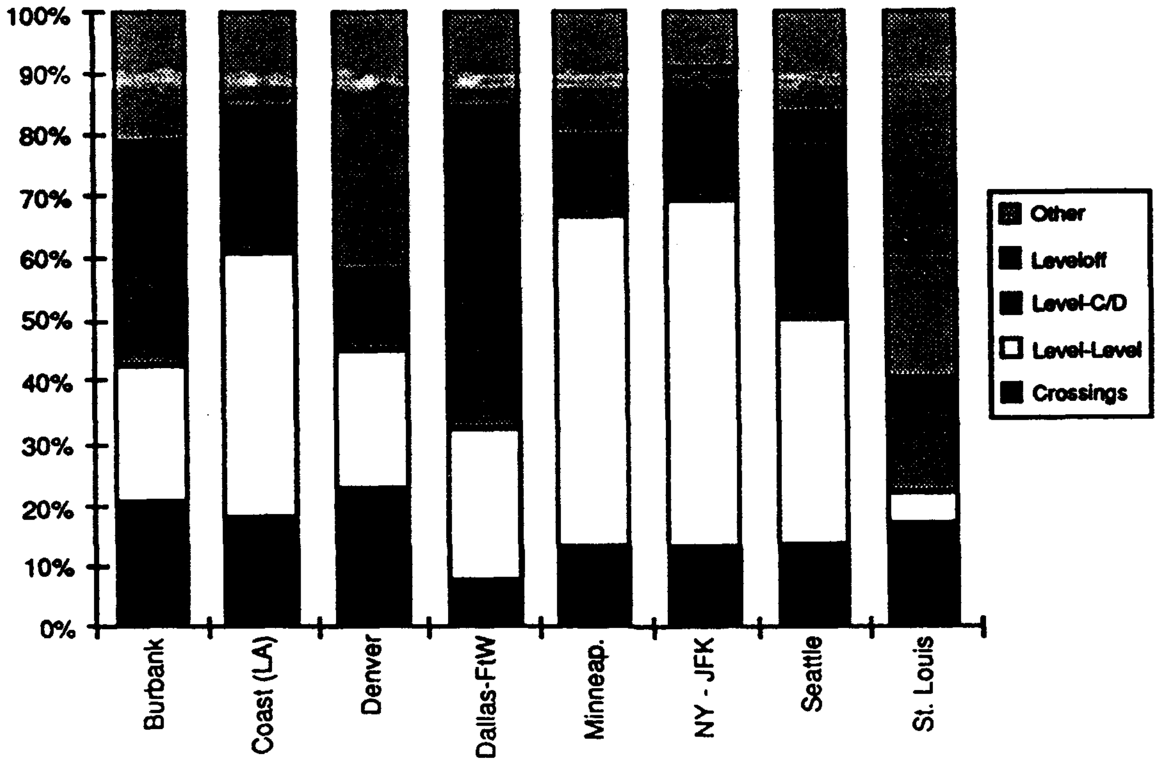



**Figure 11. Proportional Frequencies of Encounter Types by Site**

The simulations of the various encounter classes at each altitude layer produced distributions of vertical separation that were substantially equal to the intended separation. This separation, designated by the value of the logic parameter ALIM, is decreased somewhat in v6.04.

Figure 12 provides a three-dimensional view of this large body of data. Parts (a) through (f) of the figure compare the vertical separation results in altitude layers 1 through 6, respectively, for a single encounter class (class 11). Part (g) combines all these data in a single plot, showing that the greatest mass of the separation increases for the higher layers, as is the intent of the logic.

These plots are highly informative in showing that v6.04 produces marginally smaller separation, as designed, and with a very small number of encounters with small separation relative to those with normal separation. In addition, it is important to observe that these results were generated for uniform numbers of trials at every band of vertical separation

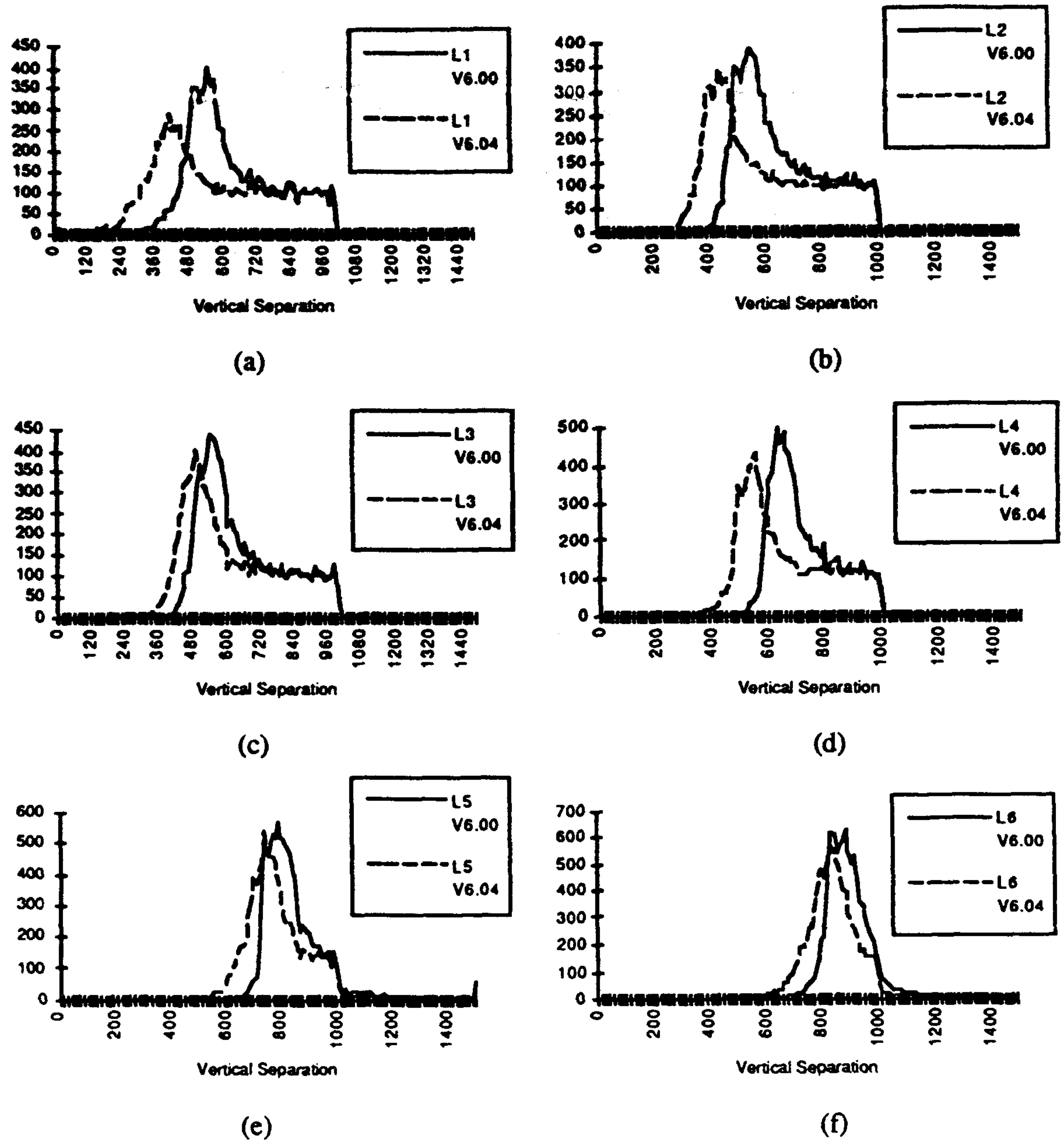



**Figure 12. Simulation Results by Altitude Layer: Perceived Vertical Separation**

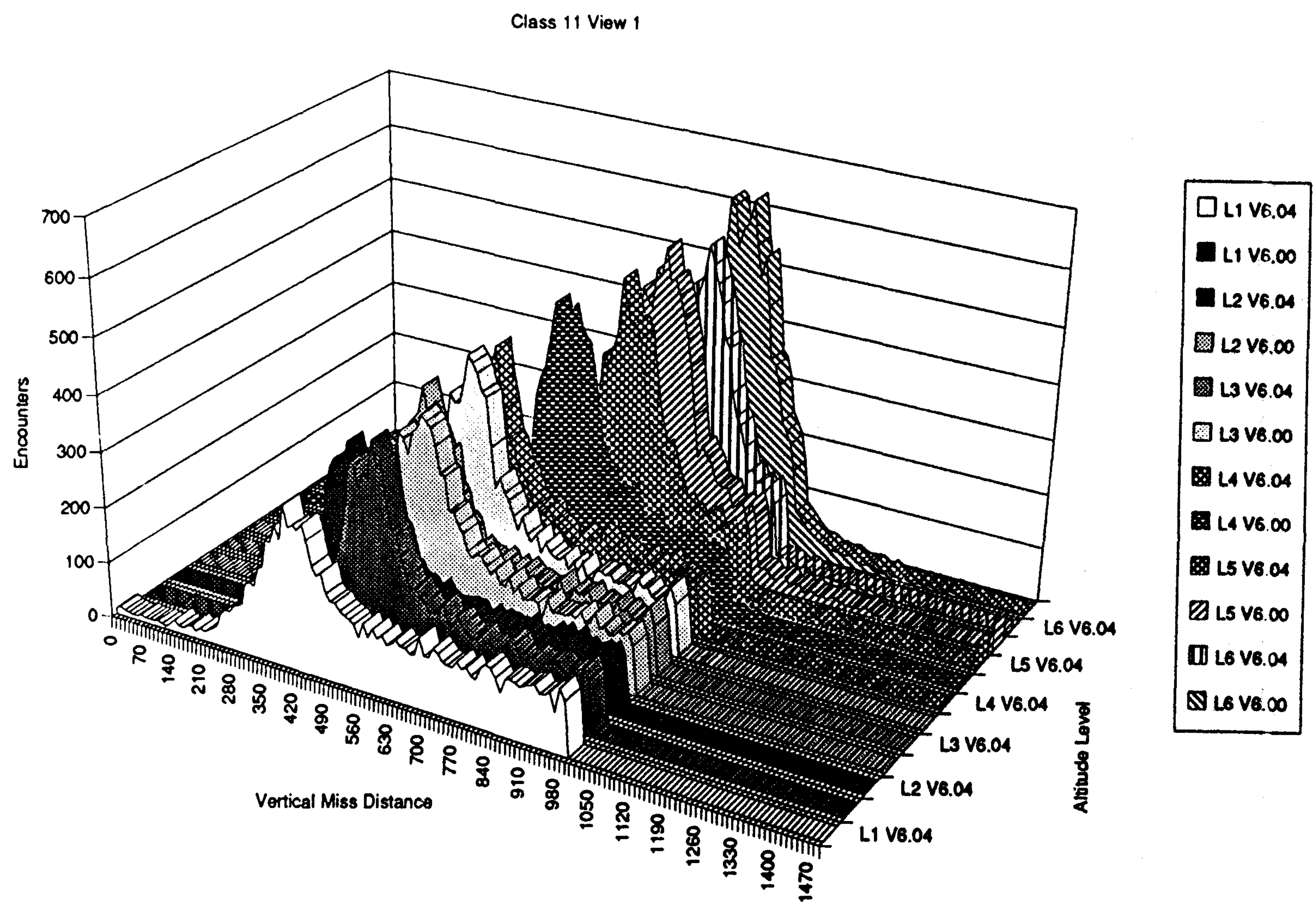



**Figure 12. Simulation Results by Altitude Layer: Perceived Vertical Separation (Concluded)**

(before TCAS acted). The risk calculations performed below combine these results in the proper proportions. This combination cannot be intuitively estimated by observing the graph alone.

The simulation results apply to apparent separation, before considering the effects of altimetry error. While these results alone are not sufficient to assess risk, they provide insight regarding the effectiveness of the logic overall, as well as for particular encounter classes.

Figures 13 and 14 show the performance of the v6.04 for encounter classes 13 and 14 respectively. The performance for the other classes simulated are shown in appendix C. These figures compare the fraction of NMACs that resulted for the runs performed at each altitude layer. The bars labeled "Unresolved" give the fraction of NMACs without TCAS that TCAS did not resolve. The bars labeled "Induced" give the fraction of non-NMACs that became NMACs after using TCAS.

For class 13, about 94 percent of NMACs were resolved in layer 1, increasing to over 99 percent in layers 2 and 3, and 100 percent in the higher layers. The induced cases represent less than 1 percent of each layer, decreasing to about 0.1 percent in the higher layers. For class 14, 97 percent of NMACs were resolved in layer 1, and 99.9 to 100 percent in the higher layers. The largest induced bar for this class represents .06 of one percent.

These figures reflect a simple summing, rather than a weighted combination, of the various VMD bands. The heights of these bars must be adjusted by weighting before a risk computation may be made. However, they are useful in comparing the results of the layers. They show that layer 1 has far more NMACs than the other layers. (Recall that equal numbers of encounters were simulated for each layer.) This salient pattern is repeated in every encounter class. The data also shows that most of the failures come from Unresolved NMACs. Only a few classes have a substantial number of Induced NMACs, and even for these classes, only a few VMD bands are susceptible.

## 3.3 CALCULATION OF COMBINED ALTIMETRY AND LOGIC PERFORMANCE

This section presents the results of the process described in section 2.5, which calculates the Risk Ratio component due to the combined effects of logic and altimetry errors. This calculation assumes all RAs are followed. The calculation is done for each of the eight sites using both the encounter class distributions and the vertical weight distributions within each class as observed at that site.[3] In addition, an average figure is developed, which is based on averaging the class weights and |VMD| across sites.

[3] As discussed in Sections 2.3.2 and 3.1.1, the class distributions and weights are independent of the version of logic.

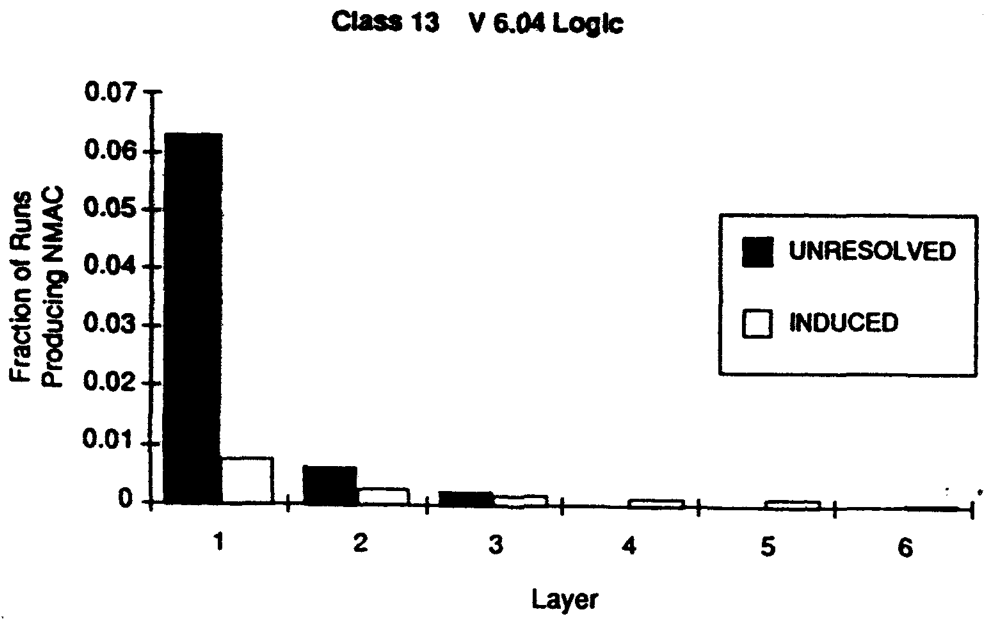



Figure 13. NMAC Statistics for Simulations of Class 13

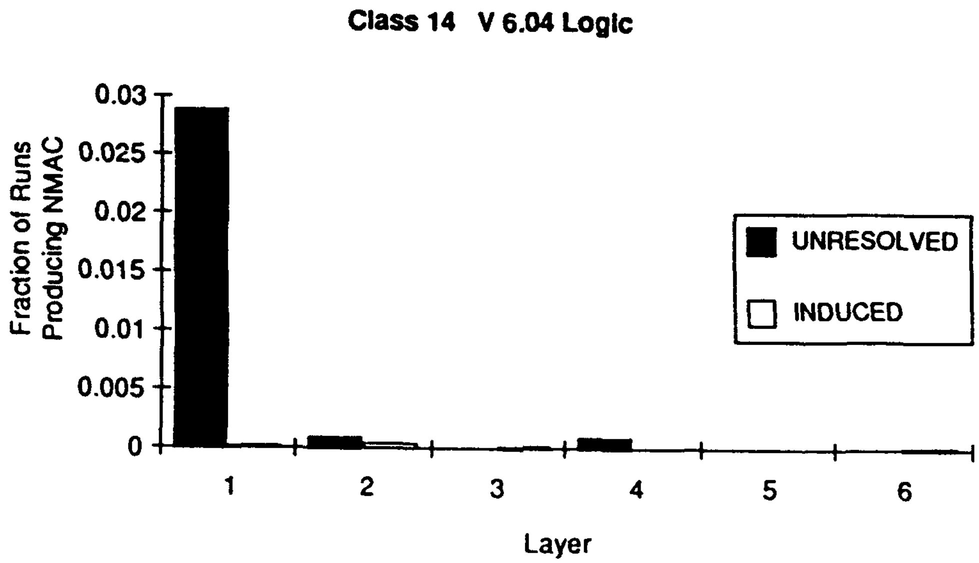



Figure 14. NMAC Statistics for Simulations of Class 14

Table 6 shows that this component of Risk Ratio increases by 2.8 percent for the "average" basis used, ranging from 0.8 percent to 4.4 percent at the eight sites. This is of the same order as the variation of the Risk Ratio among sites for the present v6.0 logic, which varies from 0.8 percent to 5.1 percent. Risk Ratios are a means of comparison to the NMAC risk prior to TCAS; the values discussed here are 1.5 to 2 orders of magnitude smaller. The Fault Tree calculations in section 3.5 make use of this component of Risk.

Since it was observed in section 3.2 that v6.04 performance in layer 1 was markedly degraded relative to the other layers, the Risk Ratio Component was recalculated for layer 1 alone, and for layers 2-6, excluding 1. The results, shown in table 7, indicate that the logic versions are considerably closer in performance for layers 2-6.

**Table 6. Risk Ratio Increment by Site**

| Site | Risk Ratio Component V 6 | Risk Ratio Component V 6.04 | Increment in Risk Ratio Component |
|---|---|---|---|
| Burbank | .0279 | .0724 | .0445 |
| Coast | .0514 | .0595 | .0081 |
| Denver | .0130 | .0377 | .0247 |
| Dallas-Ft Worth | .0181 | .0513 | .0332 |
| New York (JFK) | .0097 | .0340 | .0243 |
| Minneapolis-St Paul | .0121 | .0398 | .0277 |
| Seattle | .0129 | .0410 | .0281 |
| St Louis | .0085 | .0271 | .0186 |
| Average | .0145 | .0429 | .0284 |

**Table 7. Risk Ratio Increment by Altitude Layer**

| Layer | Risk Ratio Component V 6 | Risk Ratio Component V 6.04 | Increment in Risk Ratio Component |
|---|---|---|---|
| 1-6 | .0145 | .0429 | .0284 |
| 1 only | .0204 | .1170 | .0966 |
| 2-6 only | .0135 | .0306 | .0171 |

## 3.4 SENSITIVITY STUDIES

### 3.4.1 Imperfect Surveillance

Several encounter geometries were rerun with the probability of TCAS surveillance delivering a Mode C report set to 0.9 and 0.8 respectively. The value 0.8 is judged to be a reasonable "worst-case" probability of receiving reports through a high density interference environment. The Risk Ratio components calculated from these runs were compared with the component using perfect reply probability.

Figure 15 illustrates these results. In part (a) of the figure, crossing encounters of class 1 were run at layer 3 (7000 ft altitude). Performance degrades marginally with surveillance quality, while the increment of Risk Ratio changes from about one percent to 1.5 percent. Part (b) of the figure shows results for leveloff encounters of class 13 also run at layer 3. For these, the Risk Ratio component actually improves slightly for this marginally degraded surveillance. This result reflects larger separations being generated, possibly more than necessary in some cases. Most significant is that the increment in Risk Ratio from the version 6.04 logic closely matches in both geometries.

These results indicate that imperfect surveillance quality will not have a significant effect on the relative performance of these logic versions.

### 3.4.2 Restricted Maneuver Capability

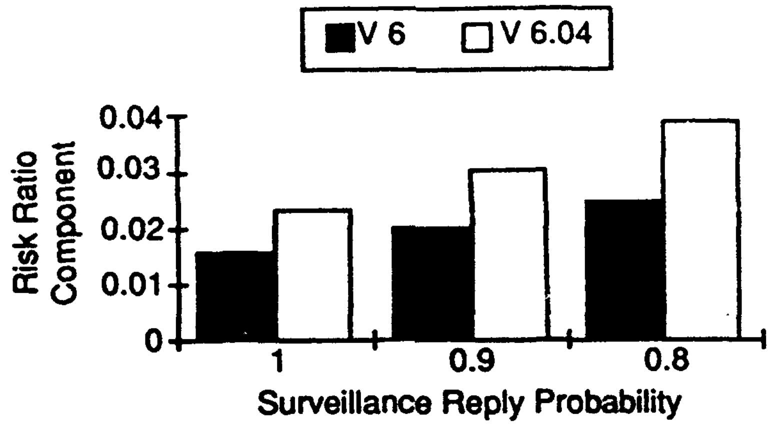
Several encounter geometries were rerun using v6.04 to study the effects of TCAS resolution when the TCAS aircraft is unable to climb or to descend. The logic takes account of these conditions when selecting an advisory.

**Climb-inhibited Condition**

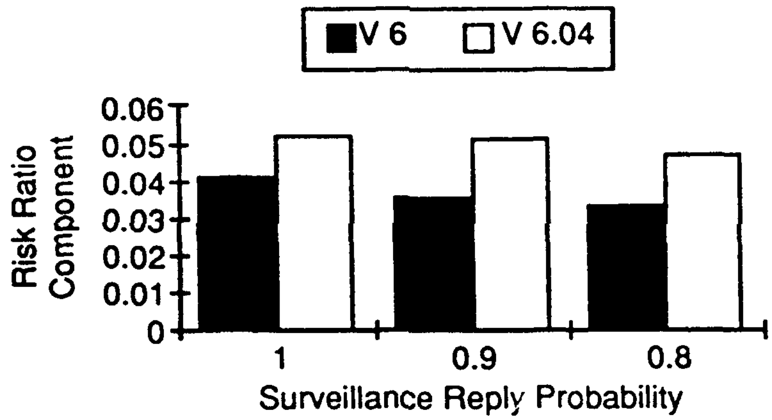
The climb-inhibited condition was simulated for several geometries in altitude layer 6, using the corresponding logic parameters. As in the normal case, each aircraft in the encounter was simulated in turn to carry TCAS.

Degraded performance was observed for geometries in which the "higher" aircraft carried the TCAS and in which the threat passed closely (e.g., 100 to 300 ft) below. Where TCAS could not achieve good separation with a crossing maneuver, and yet was unable to issue a "Climb" advisory, the existing separation often was unchanged.

The climb-inhibited condition is expected to be extremely rare. In this altitude regime, where standard vertical separations are 2000 ft, such close passages are infrequent. Furthermore, only a few combinations of aircraft and flight regimes have been identified as inhibiting TCAS Climb advisories. No measurable effect on the average risk calculation is expected.




(a) Crossing Encounters




(b) Leveloff Encounters

**Figure 15. Logic Comparison for Imperfect Surveillance**

**Descend-inhibited condition**

The descend-inhibited condition applies within a narrow altitude band in the lowest altitudes where TCAS generates RAs. The logic does not issue "Descend" advisories that would produce close proximity with the ground. Several geometries were simulated at approximately 1000 ft altitude above ground level (AGL), using the corresponding logic parameters for layer 1. The results were compared to the "normal" layer 1 simulations performed at 2000 ft.

The descend-inhibited simulations showed severely degraded results when TCAS was the "lower" aircraft in the encounter, and was only several hundred feet above the lower limit of its operating regime. In this situation, TCAS cannot issue a "Descend" advisory, and due to the short warning time at low altitudes, cannot achieve adequate separation with a nominal climb. Consequently, TCAS is often ineffective in changing the pre-existing vertical separation. It is anticipated that this effect is a strong function of the height above the descend-inhibit region.

The descend-inhibited condition only applies over a small band of low altitudes. Its effect on safety is somewhat like extending the "TA-only" mode higher for certain aircraft in certain geometries. Further work must be performed to measure the change as a function of altitude.

## 3.5 FAULT TREE CALCULATIONS

This section collects the various elements contributing to NMACs and calculates their total probability relative to the NMAC risk without TCAS. The process draws upon the original fault tree developed for the TCAS Safety Study [3], with certain changes to account for the more sophisticated analysis of logic performance.

The original fault tree (figure 16a shows a simplified version) separated NMAC events into two categories termed "Unresolved" and "Induced." These categories respectively referred to events that would have been NMACs without TCAS and which TCAS did not prevent; and those that would not have been NMACs except for following TCAS advisories. In these earlier studies, the Unresolved category included items such as threat's lack of Mode-C equipage, surveillance failure to track, and altimetry error. The Induced category included erroneous RAs due to altimetry error, stuck C bits, and maneuvering intruders.

The present study examines TCAS logic performance in a more comprehensive manner and recognizes that some of the resulting NMACs cannot be meaningfully categorized as either Unresolved or Induced. An example would be an encounter for which the maneuver in response to an RA provides 200 ft perceived separation, while altimetry errors negate this separation, resulting in true separation less than 100 ft. In this example, both logic performance and altimetry errors contributed to the NMAC; the earlier methods would not have counted it at all.

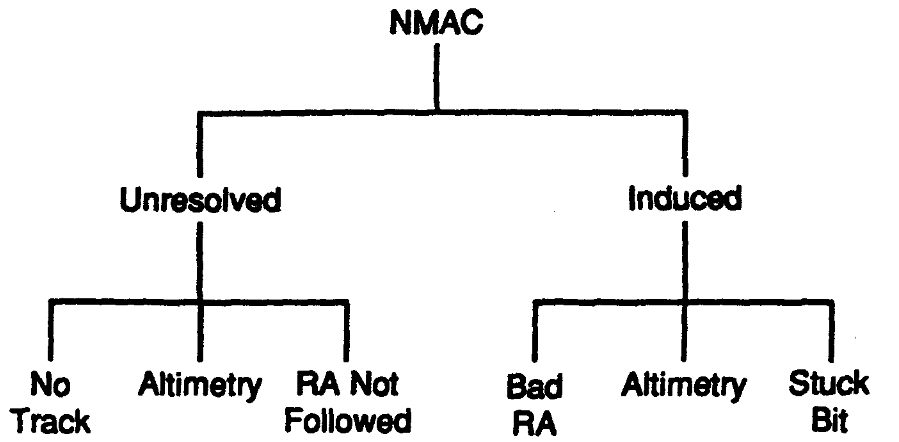



**Figure 16a. Previous Fault Tree Structure (Simplified)**

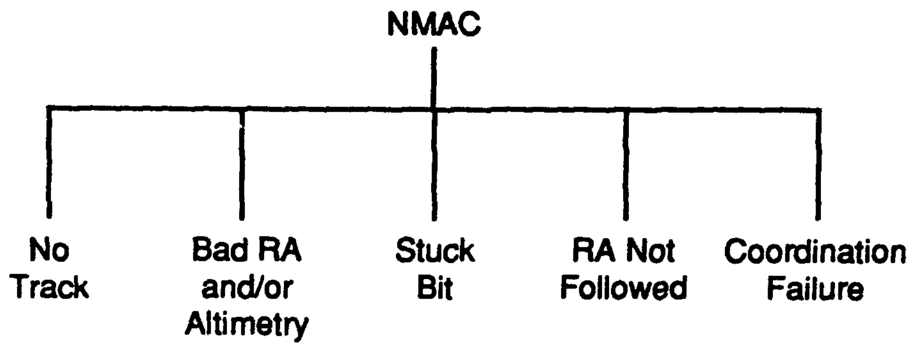



**Figure 16b. Revised Fault Tree Structure**

*Ineffective Resolution Advisories and Altimetry Errors have been combined into a single category. The distinction between Unresolved and Induced NMACs is removed for logic-related causes. The new causes of RAs not followed and of failures in TCAS-TCAS coordinated encounters have been added.*

Accordingly, this study drops the distinction between Unresolved and Induced NMACs for cases related to altimetry and logic performance, and rearranges the fault tree as in figure 16b. The new fault tree still contains the same failure paths, and clearly distinguishes those NMACs which are unresolved due to the threat's lack of Mode C equipage from NMACs resulting from altimetry and logic performance effects. Both logic versions are compared using this new fault tree.

Figure 17 shows the evaluation of the fault tree. The probability of each event is shown, with each boxed value indicating the compound probability for a chain of NMAC events. These are summarized in the tables below. Events and boxes that pertain to logic show values for each version. The chains that do not end with boxed numbers represent successful resolution.

In the following tables, the key factors and their probabilities used in the fault tree are listed, followed by the NMAC probabilities (boxed values) from the fault tree. They are totaled in the Summary section below to produce the final risk figure for the condition of all-RA's followed.

## 1. Unresolved NMAC related to threat's lack of Mode C equipage

This category is the same for both logic versions. The equipage figures are the same ones used in the 1988 study for the case called "Mode C NPRM" [4, appendix H]. The FAA subsequently adopted the rule requiring Mode C carriage in certain aircraft categories and certain airspace; therefore, these figures should now become the baseline case.

| Factor | Probability |
|---|---|
| Threat has no transponder | 0.02 |
| Threat has Non Mode C transponder | 0.10 |
| Mode C equipped | 0.88 |
| TA issued | 0.94 |
| Visual conditions permit acquisition | 0.70 |
| Visual acquisition made by 15 sec | 0.83 |

| EVENTS | RISK RELATIVE TO NO TCAS |
|---|---|
| Threat has no transponder | .02 |
| Threat has Non Mode C transponder AND TA not given | .006 |
| Threat has Non Mode C transponder AND TA issued AND visual conditions do not permit acquisition | .0282 |
| Threat has Non Mode C transponder AND TA issued AND visual conditions good AND no visual acquisition made by 15 sec | .0112 |

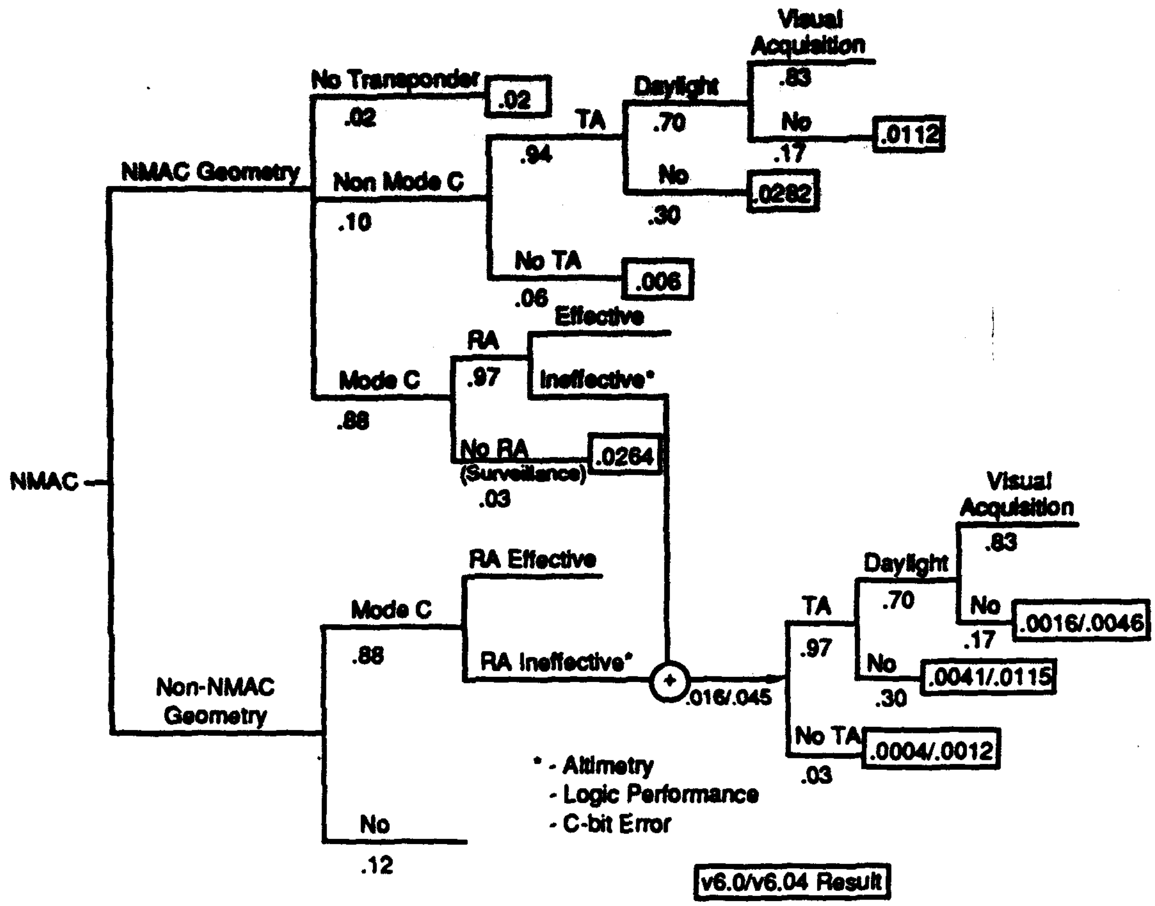



**Figure 17. Evaluation of Fault Tree**

### 2. Unresolved NMAC related to Surveillance limitations

This category is the same for both logic versions. The contributors are the Mode C equipage and TCAS surveillance failure to track a threat so that a timely RA is not issued. The surveillance failure refers to the failure to issue a timely RA against a Mode C threat (not Mode S), and intentionally overstates the probability of failure because this probability decreases as the range to a collision threat decreases. This would create an increased chance to resolve the encounter with a late RA, but no such credit is taken here.

| Factor | Probability |
|---|---|
| Mode C surveillance fails to track threat | .03 |

| EVENTS | RISK RELATIVE TO NO TCAS |
|---|---|
| Mode C threat AND no RA issued due to surveillance failing to track threat | .0264 |

### 3. NMAC related to altimetry and logic performance

This category applies the results of the simulation of logic performance and altimetry errors to the same factors used in the fault tree of the 1988 study. The figures given for Logic and Altimetry effects represent the site averages, and are based upon TCAS RAs always being followed.

| Factor | Probability | | | |
|---|---|---|---|---|
| TA alerted pilot prior to RA being issued | .97 | | | |
| Visual acquisition by 15 s before CPA | .83 | | | |
| Logic and altimetry give < 100 ft separation | .014 | (v6.0) | .043 | (v6.04) |
| C-bit error causes < 100 ft separation | .002 | | | |

| EVENTS | RISK RELATIVE TO NO TCAS | |
|---|---|---|
| | v6.0 | v6.04 |
| Small separation AND no TA given | .0004 | .0012 |
| Small separation AND TA given AND visual conditions do not permit acquisition | .0041 | .0115 |
| Small separation AND TA given AND visual conditions good AND no visual acquisition made by 15 sec | .0016 | .0046 |

**Summary**

| | v6.0 | v6.04 |
|---|---|---|
| Unresolved NMAC related to non-logic factors (Sum of Parts 1 and 2 above) | .0918 | .0918 |
| NMAC related to altimetry and logic (Sum of Part 3 above) | .0061 | .0173 |
| TOTAL | .0979 | .1091 |

These results show that logic-related NMACs are small compared to residual risks not related to the logic. Figure 18 illustrates the relative proportions. The change to v6.04 has little impact on the total risk. The contribution of non-logic related factors (Mode C equipage, surveillance, visual acquisition) dominates the altimetry and logic-related factors. The result shown, based upon the site average figures, gives an increment of approximately one percent of the no-TCAS risk. This result is fairly consistent for all the sites studied: the largest site increment is 1.7 percent. Obviously, even a small degradation in protection would not be recommended were it not more than compensated by the benefits sought.

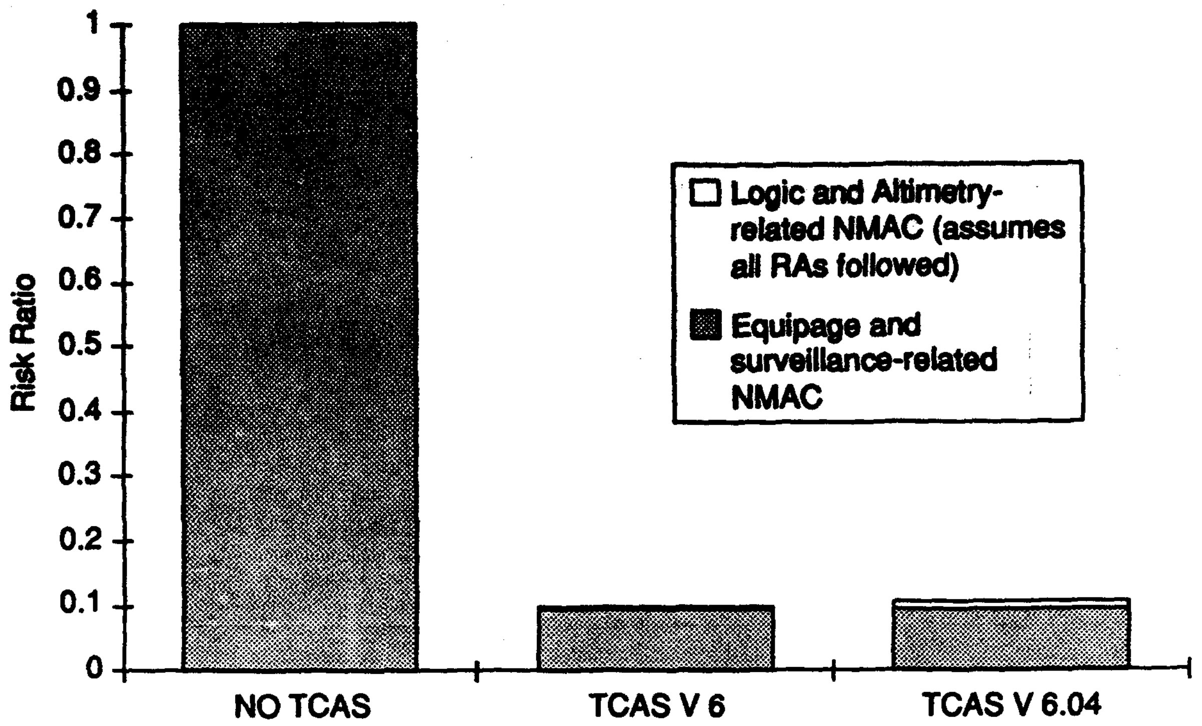



**Figure 18. Risk Ratio Components**

If the result were recalculated for altitude layers 2-6 only, using the data from table 7 above, the one percent difference would become 0.65 percent. The logic differences are quite small in these layers, the greatest part of risk increment being contributed by layer 1.

Of the small part of Risk Ratio that is logic-related, only a fraction is TCAS-induced, section 3.2 showed that unresolved NMACs form the majority of the NMAC encounters seen in simulation.

It must be emphasized that these calculations are based on the condition that all TCAS RAs are followed except those for which the pilot can recognize that to follow the RA would be unsafe. The next section makes important observations that modify these results for more realistic conditions related to the frequency of following RAs.

## SECTION 4

## SPECIAL ANALYSES

**This section contains analyses of the two major branches of the Fault Tree that were not included in the modeling and simulation process described in sections 2 and 3. Section 4.1 analyzes the effects on safety of pilots not complying with their TCAS RAs. Section 4.2 analyzes the safety aspects of TCAS-TCAS coordinated encounters.**

### 4.1 EFFECTS OF NOT FOLLOWING RESOLUTION ADVISORIES

The simulation analysis described above applies to conditions where TCAS RAs always are followed, unless visually recognized as unsafe to do so. However, early operational experience using v6.0 of TCAS has shown the need for a better match with normal ATC operations, and that some significant fraction of TCAS RAs are not followed. This section discusses the implications of such actions on calculating TCAS safety. Observations also are given below regarding the potential for improving the realized level of safety with the new logic v6.04.

Figure 19 is a conceptual illustration of the variation of average NMAC risk according to the fraction of TCAS RAs that are followed. This analysis assumes that this fraction is the same for all "close" TCAS encounters that represent NMAC risk. Also, it does not apply in the special circumstance of TCAS-TCAS coordinated encounters, which are discussed separately.

The variable plotted along the x-axis represents the fraction of RAs followed, excluding any cases for which the pilot can visually recognize that the RA would be unsafe to follow (for example, excessive altimetry error). For all the other RAs, the pilot cannot recognize any as unsafe, but might not follow the RA for various reasons. These causes may include inattention, distraction, incompatibility with own intentions, incompatibility with ATC clearance, pilot's selection of another resolution, or recognition that the encounter will achieve safe separation without a maneuver. However, in the context of Risk Ratio, only NMAC encounters or other close encounters are considered. "Nuisance" encounters that are clearly safe are not included; RAs that are not followed for such encounters do not apply (except possibly to influence pilot tendencies).

The figure shows, as more RAs are followed, TCAS provides a successively increasing level of effectiveness, with former NMACs resolved, and a corresponding decrease in the remaining risk. Although the logic and altimetry-related NMACs increase, their frequency is considerably lower than the non-logic related unresolved NMACs.

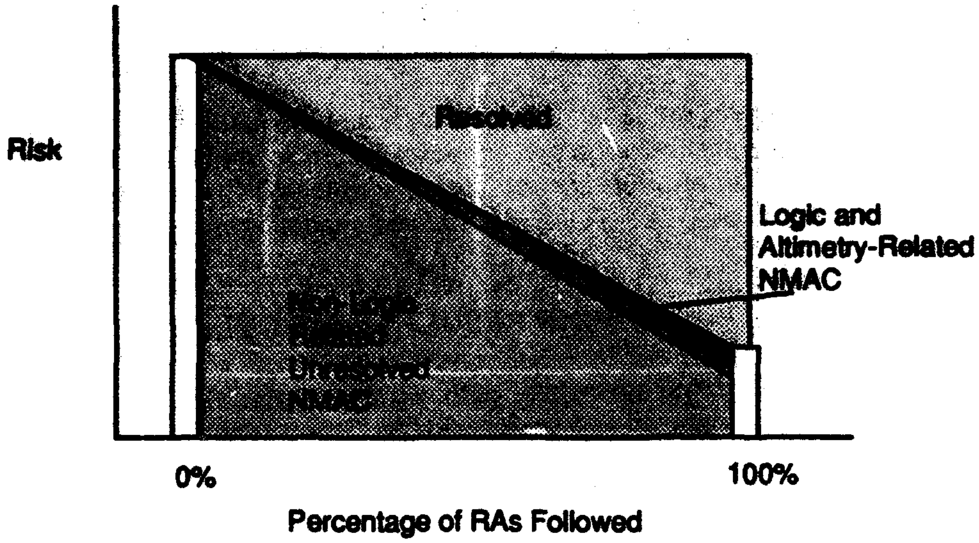



**Figure 19. NMAC Risk According to Fraction of RAs Followed**

Figure 20 is another conceptual illustration comparing the relative risk without TCAS to the remaining risk upon using TCAS v6.0 and v6.04 when all RAs are followed for each version. (The figure is not drawn to scale. Figure 18 above shows the correct proportions.)

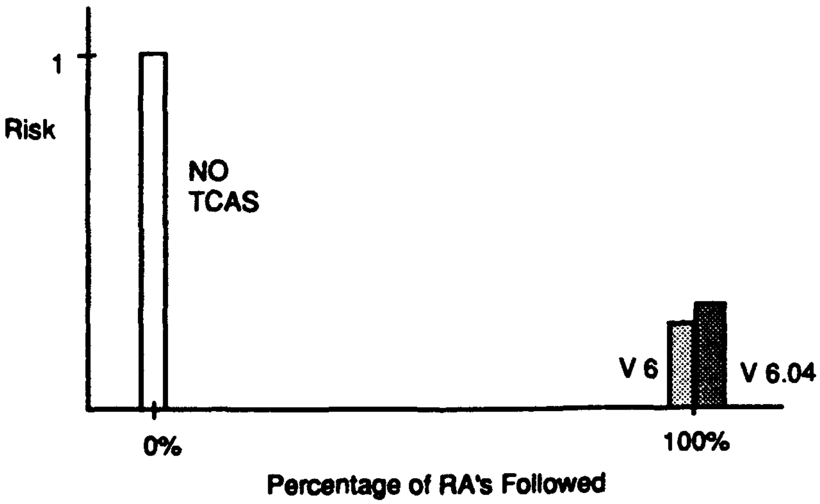



**Figure 20. Relative Risk for 100 Percent of RAs Followed**

Figure 21 overlays the continuous variation from figure 19 onto the endpoints of figure 20. These lines compare the decrease in Risk for the two versions of logic when treating the fraction of RAs followed as a continuous variable. The scale is exaggerated for clarity in illustrating the following concept. At any constant value of RAs followed, v6.04 has higher risk; however, observe that for the point where "X" percent of RAs are followed using the present (v6.0) logic, there is a corresponding point labeled "Y" percent where the v6.04 logic has identical Risk. Since the 6.04 logic is intended to eliminate most nuisance alerts, it is anticipated that TCAS advisories will be followed more frequently using v6.04. Therefore, it would require only a small increase in this rate to achieve a lower risk in practice than is now being achieved using v6.0.

Figure 22 shows the actual plot of the site average Risk variation, drawn to scale. This figure uses as its endpoints the Risk Ratio result that was computed in section 3.5 for the condition of all aircraft following their advisories. The two curves are extremely close together, reflecting the dominance of non-logic contributors to risk, principally non-transponder and non-Mode C equipped aircraft, and TCAS Mode C surveillance limitations. For these curves, even a one percent increase in the percentage of RAs followed would improve the overall risk. A five percent increase in the percentage of RAs followed would decrease overall risk by about four percent. Even if the logic-related portion of these curves were somehow nonlinear, the variation would be much less than four percent.

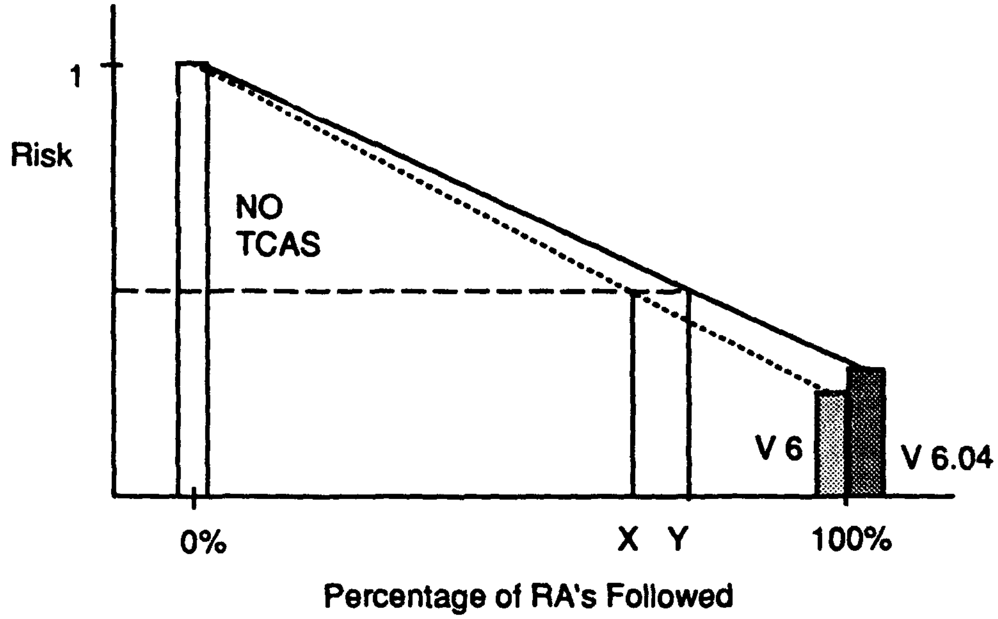



**Figure 21. Risk Comparison With Different Compliance for Alternate Logic Versions**

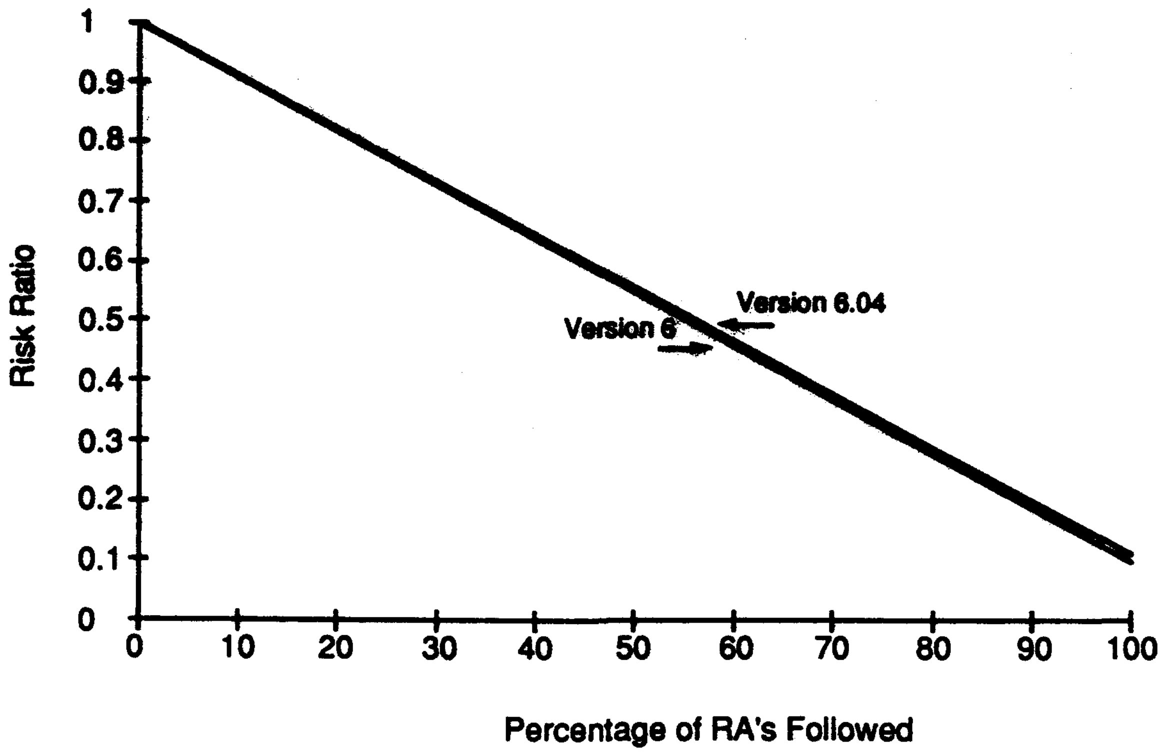



**Figure 22. Risk Variation According to Fraction of RAs Followed—Site Average Risk**

## 4.2 COORDINATED ENCOUNTERS

The conventional approach to TCAS Safety treats the risk as negligible for those encounters where both aircraft are TCAS-equipped. Since the logic ensures the selection of coordinated RAs, the risk of NMAC should be extremely remote. This section examines second-order effects and develops a more tangible estimate of this risk.

In a TCAS-TCAS encounter, an NMAC could result if any of the following were to occur:

1. One TCAS failed to track the other AND another failure chain occurred as in a TCAS vs. non-TCAS encounter.

Some of the subcases in this category include:

- One TCAS failed to track the other AND maneuvered to thwart the other TCAS' escape maneuver. (This failure is unique since the logic does not issue reversals against a TCAS threat.)

- **Both TCAS fail to track the other and the geometry leads to an NMAC.**

2. **The Mode S Air-Air Coordination Link failed AND both TCAS units displayed incompatible advisories (such as CLIMB for both aircraft) AND the incompatibility was not detected and reversed in a timely way AND the geometry was such that these maneuvers could cause a NMAC.**

3. **After successful coordination, the pilot of one aircraft followed a TCAS advisory while the pilot of the other aircraft failed to respond to the TCAS advisory or selected a maneuver incompatible with the first TCAS' resolution.**

4. **The two TCAS units coordinated normally, both RAs were followed, and the resulting maneuvers brought about a NMAC.**

The following subsections address these situations.

### 4.2.1 Surveillance Failure

MIT Lincoln Laboratory has studied the Mode S surveillance results of flight testing and determined [10] that for a Mode S threat with diversity antennas (as a TCAS II threat would have), the probability of track acquisition would be at least .999 and the subsequent probability of receiving each updated report would be at least .99. These figures apply within a 12 nmi range, and improve considerably with range closure. They apply only to the link reliability, and do not consider aircraft that cannot be tracked because of an illegal Mode S address, or an address duplicating TCAS' own address—such a probability should become remote, and is difficult to estimate.

With this probability of a surveillance failure, the probability is of the order $10^{-3}$ or lower that either TCAS would fail to track the other, or would lose track before issuing an RA. This probability multiplies the various risks that apply to encounters against non-TCAS threats (except, of course, the Mode C surveillance failure). Also, the probability is of the order $[10^{-3}]^2 = 10^{-6}$ that the encounter would be unresolved due to both TCAS aircraft independently failing to track each other.

The probability of Mode S surveillance failure is thus negligible relative to the .03 probability used for Mode C surveillance failure. Accordingly, this event should not contribute measurably to the Risk Ratio.

### 4.2.2 Coordination Link Failure

The second of these event chains has been evaluated by Lincoln Laboratory [11]. Extracting the relevant portions of their analysis, we calculate:

| | | |
|---|---|---|
| | P (link dropout that prevents coordination before advisory display) | .000012 |
| x | P(link dropout prevents timely selection of compatible sense) | .00000011 |
| x | P(inadequate horizontal separation) | .08 |
| | | $1 \times 10^{-13}$ |

This result states that one in ten trillion TCAS-TCAS RAs would be expected to have both uncoordinated vertical sense and inadequate horizontal separation. This is smaller by many orders of magnitude than most other risks considered in this study. It could be reduced even more by considering that for only a fraction of such encounters would both TCAS aircraft select the same (incompatible) sense, even without coordination.

### 4.2.3 Failure to Follow TCAS Advisory

In a TCAS-TCAS encounter, the logic and coordination functions ensure selection and display of compatible advisories. The FAA Technical Center has performed extensive simulations of such encounters [12] and have concluded that in virtually every encounter geometry, the two aircraft could have avoided a NMAC by following their respective TCAS advisories. This leads to the conclusion that a maneuver contrary to the direction advised by TCAS is likely to reduce the vertical separation. A simple failure to respond (rather than a maneuver in the direction opposite to the advisory) may have various results: the other TCAS may achieve good separation with its own maneuver, as in an encounter with a non-TCAS; however, for some geometries, one aircraft's maneuver would provide most of the resolution, while the other receives a complementary advisory primarily to preclude the opposite direction maneuver (which could negate the first TCAS' resolution). In this latter case, one aircraft's failure to respond would be of greater significance than the other's.

This failure category has potentially greater significance than any other failure studied for the TCAS-TCAS encounters. Unfortunately, its magnitude is difficult to measure and especially difficult to predict. There is no evidence that the change of logic version would increase risk of this type. To the contrary, if v6.04 promotes increased pilot confidence in TCAS, as is intended, more of its advisories may be followed, with a resulting safety benefit.

### 4.2.4 Unsafe Resolution Advisories

A coordinated TCAS-TCAS encounter should always produce a successful resolution if the advisories are followed. The FAA Technical Center has conducted extensive simulation of encounters to search for either routine or extreme conditions which could bring about an unsuccessful resolution [12]. Both versions of TCAS logic were tested, as well as

interoperability testing of a v6.0 TCAS against a v6.04 TCAS. The only geometry that was reported to fail involved an aircraft initially climbing at a rate of 5000 fpm and encountering a level aircraft as it began to level off with a 1/3 g acceleration. Both logic versions are susceptible to this geometry, although their thresholds of vulnerability differ.

In the U.S. database containing over 10,000 observed encounters, only one of the over 20,000 aircraft had a vertical rate of this magnitude during an encounter window. None of the RA-producing encounters had an aircraft with such a high vertical rate. While this data seems to exaggerate the scarcity of flight at high rates, it is clear that close encounters of this type are extremely infrequent in comparison to the routine encounters that contribute to the Risk Ratio component for unequipped intruders. The logic-related component of Risk Ratio then should be smaller by several orders of magnitude than the performance of the logic in encounters with unequipped threats.

### 4.2.5 Summary

The discussion in this section shows that none of the TCAS-TCAS events should increase the Risk Ratio except the case of the failure to follow a TCAS advisory. The factors are not explicitly logic-dependent, although any increased confidence in TCAS, as is the intent of v6.04, may bring about increased compliance.

## SECTION 5

## CONCLUSIONS

This study has examined over 10,000 aircraft encounters at eight sites having different environments with respect to encounter geometries. Encounter modeling was conducted based upon 1889 RA-producing encounters. 780,000 Simulation runs using the complete logic have exercised a wide variety of geometry types for unequipped threats.

1. For the condition that all TCAS RAs are followed unless recognized as unsafe, logic v6.04 would produce a Risk Ratio only about one percent greater than for v6.0, on a theoretical site-average basis.

2. The greatest contribution to the v6.04 increment comes from the altitude layer below 2350 ft, where the lowest warning time is used. Since the ATC system is highly structured in that airspace, using the overall distributions of encounter classes and vertical rates may be unrealistic and give pessimistic results. Excluding the lowest layer, the Risk Ratio increment is about 0.6 percent.

3. The variation of this Risk Ratio increment among the sites studied was not great, despite very substantial differences in the encounter geometry proportions that were found. The greatest change in Risk Ratio for any of these sites was 1.7 percent. This gives confidence that studying other locations also would yield results very similar to the average figure. Furthermore, the increment due to the new logic version is of the same order as normal site-to-site variations.

4. Recognizing that today pilots frequently do not follow RAs, often because of low confidence in TCAS, the achieved level of safety may be far from the ideal. If v6.04 raises pilot confidence to the point where even a few percent more RAs are followed, the achieved level of safety would increase, more than compensating for the reduction in warning time that eliminates many unnecessary RAs.

5. For coordinated TCAS-TCAS encounters, the logic, surveillance, and coordination functions are extremely safe. The greatest hazard in this situation would be the failure to follow RAs. This may be alleviated if v6.04 brings about increased compliance.

6. The Risk Ratio component due to logic is only slightly degraded by imperfect surveillance quality. The relative performance of the two logic versions appears unchanged.

7. When TCAS is in a climb-inhibited flight regime, or is descend-inhibited due to its proximity with the ground, its performance is significantly restricted. Such situations should occur very infrequently relative to the rate of close encounters addressed in this study.

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## APPENDIX A

## ALTIMETRY ERROR ANALYSIS

This appendix reports the results of a study of altimetry error on Traffic Alert and Collision Avoidance System II (TCAS II) Safety. It is an update of the analysis performed in the two preceding TCAS Safety Studies. As in these earlier studies, barometric altimetry error is considered as a potential cause of either:

1. TCAS failing to resolve an Near Mid-Air Collision (NMAC) which would have occurred in its absence.

2. A maneuver made in response to an TCAS advisory leads to ("induces") an NMAC, which otherwise would not have occurred.

These conditions can result when the algebraic sum of altimetry errors in the pair of aircraft in the encounter tend to distort the Mode C reports, and hence the perceived vertical state of the encounter geometry, to the extent that TCAS erroneously perceives safe separation in the first case, or erroneously perceives the situation and issues advisories that tend to reduce separation in the second case. (It should be noted that the same erroneous perception would be made by a TCAS onboard either aircraft, as well as by Air Traffic Control [ATC].)

The 1988 Safety Study incorporated a model for altimetry error based on the results of field studies conducted at both low and high altitudes. Different distributions were derived to represent high-quality corrected altimetry typical of air carriers, and uncorrected altimetry as found on much of General Aviation (GA). The Air Carrier distribution derived in the 1983 Safety Study combined error budgets for Static Source, Transducer, and Quantization errors, resulting in standard deviations ranging from 45 to 95 ft according to altitude. For GA, the distribution is modeled as a Laplacian (Double Exponential) function in order to conservatively upper-bound the distribution tails, with standard deviation varying from 67 ft at low altitude to 105 ft at high altitude.

The Altimetry analysis uses an analytical approach to determine the contribution to Risk Ratio. No encounters are explicitly modeled with the logic; instead, the logic is assumed to provide the nominal separation for which it is designed. Two components are considered: (1) actual vertical separation (projected ahead to closest point of approach [CPA]) and (2) errors in reported altimetry, combined for the two aircraft. These elements are considered to form a plane space. Regions in this space are identified for which these components combine so as to cause TCAS either to accept the perceived separation and fail to resolve an NMAC, or to issue an advisory which appears to increase the reported separation, but which actually induces an NMAC. The calculation considers the perceived separation achievable by TCAS according to the version of logic and the parameters in effect for each altitude layer. Version 6.04 logic has smaller TAU values, and correspondingly smaller vertical separations. Also, for potential crossing encounters, TCAS selects the non-crossing sense when it predicts that

altitude limit (ALIM) ft separation can be achieved. To conservatively upper-bound the risk, only ALIM ft separation is assumed for all of the fraction of encounters with crossing geometries.

The pertinent logic variable in this analysis is the parameter ALIM, which has been revised in the v6.04 logic. The calculation is performed over a number of altitude layers to account for: the altimetry error modeled as a function of altitude, for changing values of ALIM, and for the historical proportion of NMAC encounters reported by altitude. The study combines 79 percent of threats expected to have uncorrected altimetry with 21 percent expected to have corrected altimetry. Calculations are repeated for threats with corrected and with uncorrected altimetry models. Also, simulations of recorded encounters show that the version 6.04 logic selects a crossing sense for only four percent. The composite results follow:

| | Version 6.0 | Version 6.04 |
|---|---|---|
| Unresolved NMAC | .006 | .019 |
| Induced NMAC | .017 | .022 |
| TOTAL | .023 | .041 |

The fundamental concern motivating this study was to evaluate the decreased protection against altimetry errors that might result from the smaller values of ALIM used in v6.04. These results apply to the conditional probability of a conflict with a transponder-equipped threat. If the pilot were to follow every Resolution Advisory (RA) without visual avoidance of hazardous conflicts, the new logic would increase the risk by less than two percent on an absolute basis.

## APPENDIX B

## SIMULATION PARAMETERS

This appendix contains data for some aspects of the simulation runs described in this study. Additional documentation for the simulation will be found in [8].

### B.1 RATE DISTRIBUTIONS

Distributions for vertical rates were derived from the database. Figures 23 through 27 display the histograms of these distributions. For any encounter run, each aircraft profile used an independently selected rate from the appropriate distribution.

In these figures, the x-axis label denotes the upper limit of the histogram bar. In figure 23, absolute values are plotted, although the rate may have climbing or descending sense. For these classes, the "level" aircraft are usually nearly-level, with low rates. The class definitions limited this class to 400 feet per minute (fpm) or less. The data shows most of the aircraft to have very low average rates.

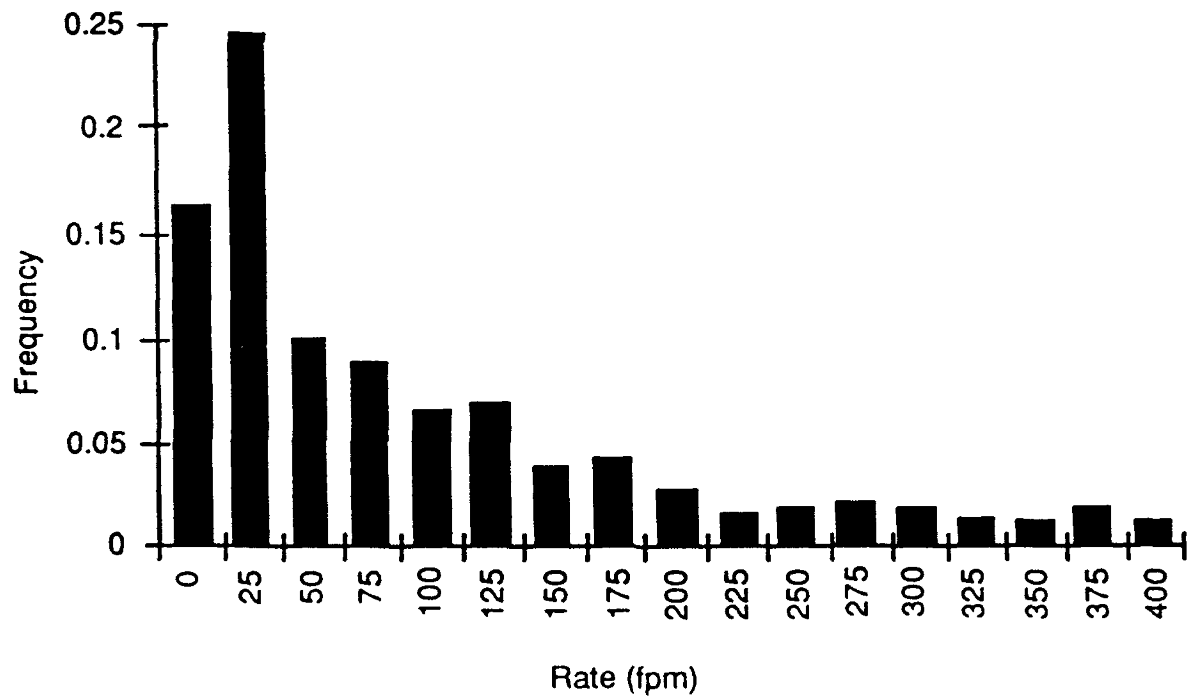


Figure 23. Vertical Rates for "Level" Aircraft

Figure 24 shows the rates applied to constant-rate aircraft and to aircraft with rates before any leveloff. Again, absolute values are plotted for these data. These rates also tend towards the lower end of the scale, although rates up to 4000 fpm are observed.

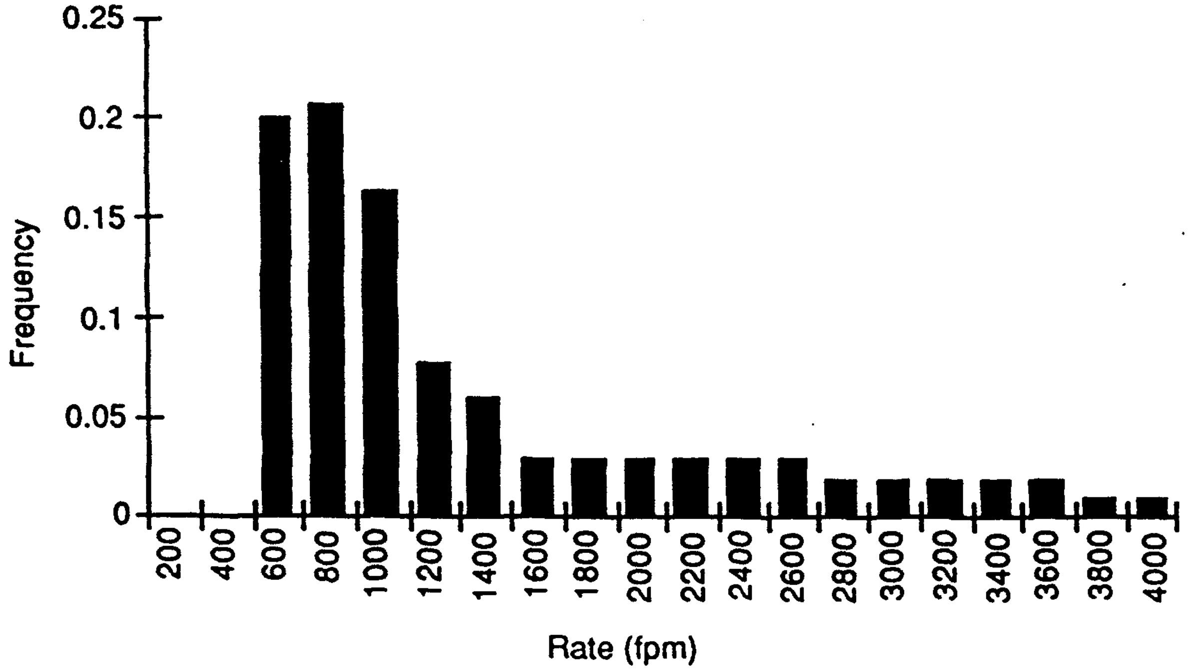


**Figure 24. Vertical Rates for Climbing or Descending Aircraft Classes 1/11, 3/13, 6/16 (Before Leveloff)**

Figure 25 displays the rates for constant-rate aircraft in different classes. These rates cluster around the level rates, which by definition do not fall within these classes. The signed rates are seen to be non-symmetrical.

Figures 26 and 27 show rates for aircraft that maneuver after initially flying level. The data, which also is signed, shows more descents than climbs, and contains mostly low rates. However, a few substantial descent rates are observed.

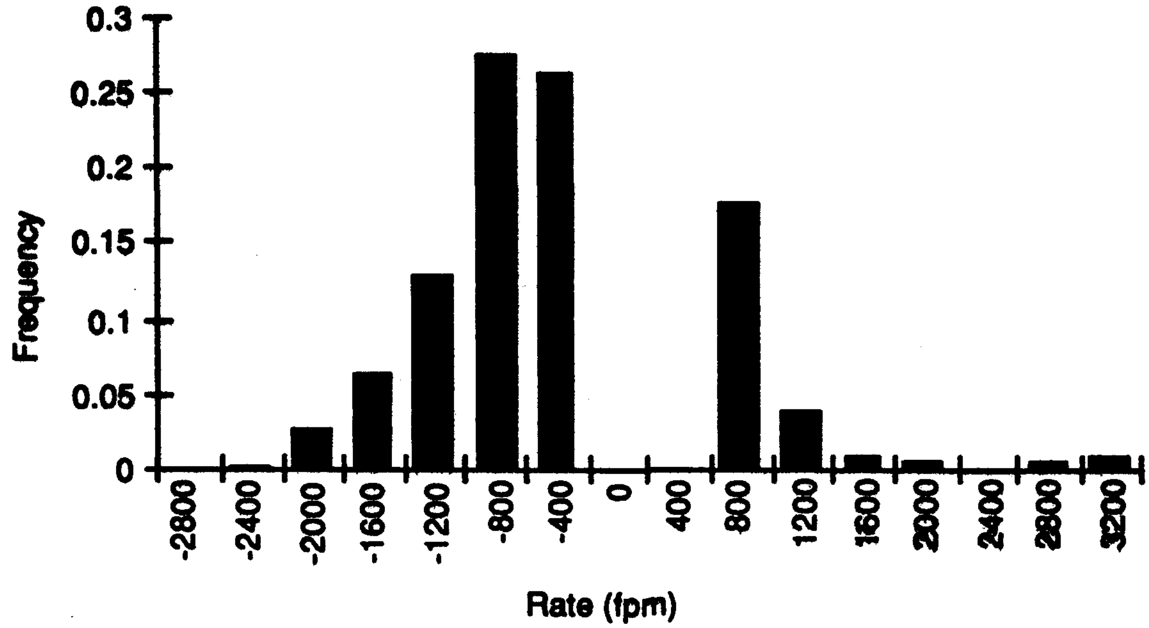


**Figure 25. Vertical Rates for Climbing or Descending Aircraft Classes 4/14, 5/15, 6/16 (Constant Rate)**

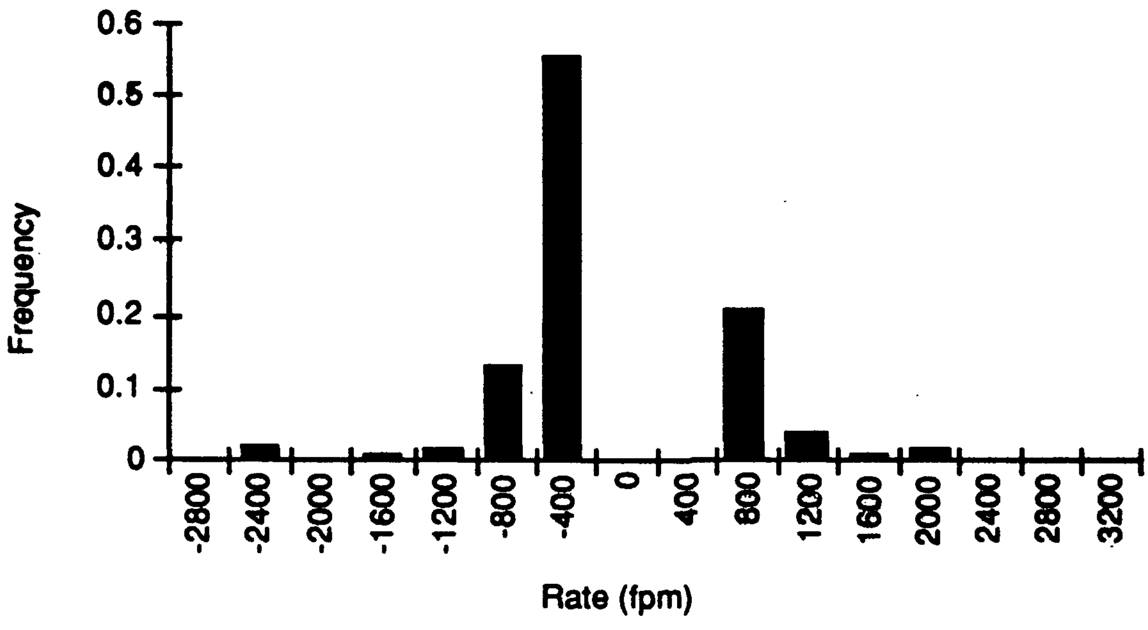


**Figure 26. Vertical Rates after Level Segment Classes 2/12**

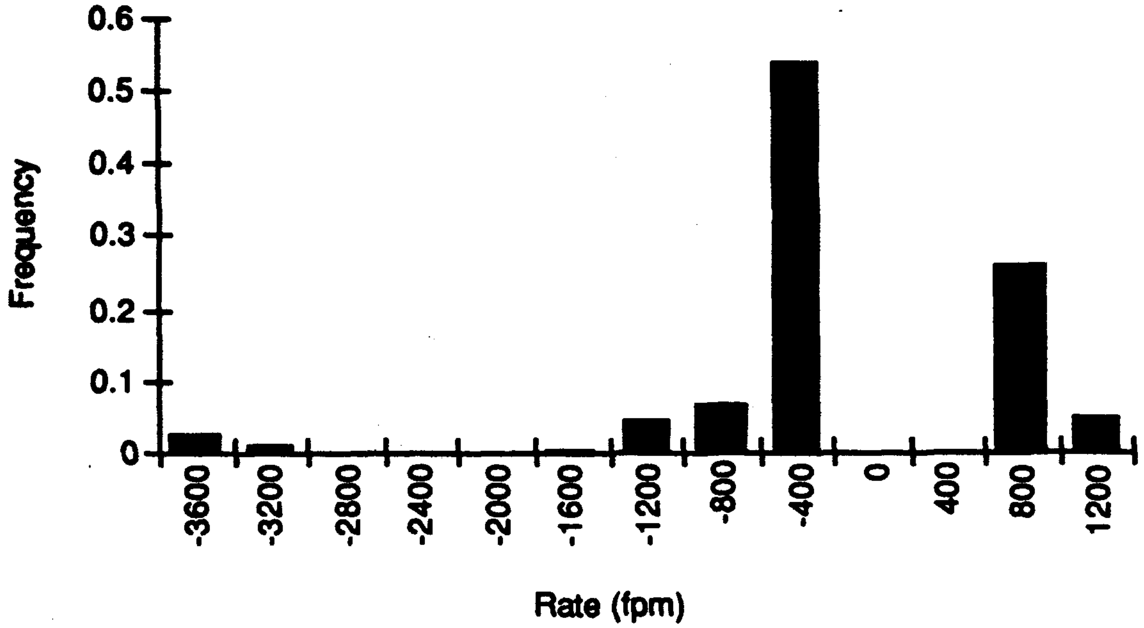


**Figure 27. Vertical Rates after Level Segment Classes 5/15**

## B.2 ACCELERATION DISTRIBUTION

The reconstructed Automated Radar Terminal System (ARTS) data does not contain enough fidelity to accurately measure accelerations; this is due to the quantization of the original altitude reports to 100 ft and to the quantization of time samples to one second. These factors make it possible only to bound the accelerations observed.

A distribution developed for simulation testing prior to the database with the help of members of the air transport community was examined for this study. This distribution recognized the frequent usage of autopilots and the consequent low values of acceleration for many maneuvers. The shape of the average rate changes in the encounter database also fit the shape of this distribution. Consequently, it was adopted for the simulation. Figure 28 shows this distribution.

## B.3 LAYER WEIGHTS

Table 8 contains the weights used to combine results from the six altitude layers. These frequencies are taken from the altitudes of reported NMACs in the Federal Aviation Administration (FAA) database that was used in the original TCAS Safety Study. The low-altitude NMACs were further divided by altitude to develop the weights for layers 1 and 2.

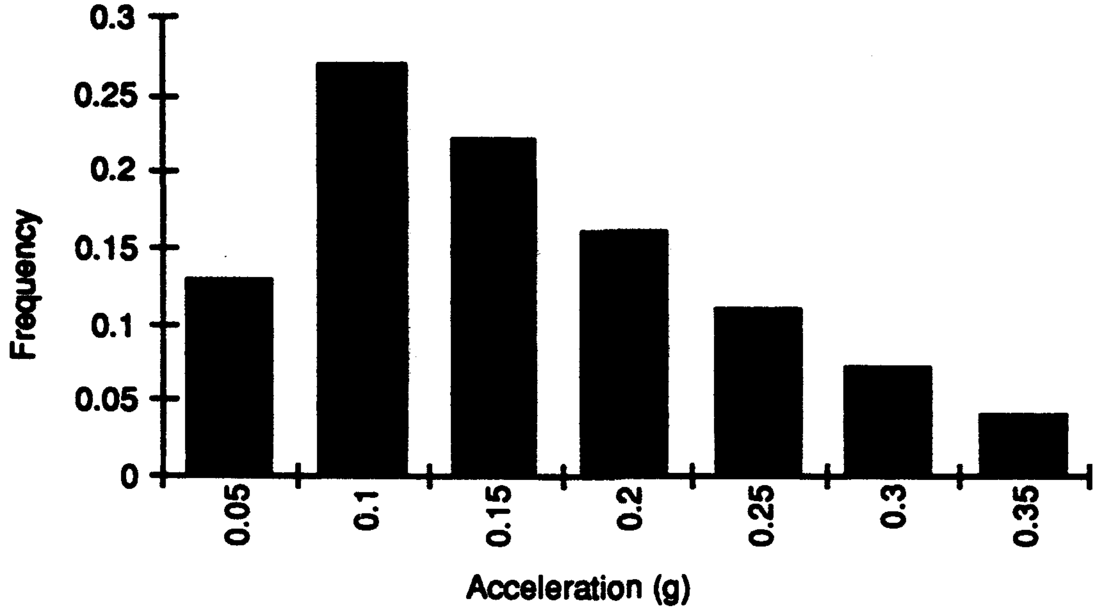


**Figure 28. Distribution of Vertical Accelerations for Simulations**

**Table 8. Layer Weights**

| Layer | NMAC Frequency |
|---|---|
| 1 | .14 |
| 2 | .27 |
| 3 | .33 |
| 4 | .21 |
| 5 | .04 |
| 6 | .01 |

# APPENDIX C

## SIMULATION RESULTS FOR NMAC

This appendix presents the data from the encounter simulations of v6.04 for every class and layer. The statistics shown give the fraction of simulated NMACs that were unresolved by TCAS and the fraction of non-NMACs for which an NMAC was induced by TCAS. These do not include the effects of altimetry error.

The fractions shown were formed by simply summing the runs for each vertical band. Post-processing, not included here, combines results of these bands by weighting them in the proportion observed in the airspace; therefore, this data is not a direct estimate of probability of these events in the airspace. However, it is a good means of comparing performance between the altitude layers.

The figures show that the unresolved component is dominant for most classes at most altitude layers.

**Table 9. V6.04 Sim Results**

| CLASS 10 | | |
|---|---|---|
| LAYER | UNRESOLVED | INDUCED |
| 1 | 0.034 | 0.00011 |
| 2 | 0 | 0 |
| 3 | 0 | 0 |
| 4 | 0 | 0 |
| 5 | 0 | 0 |
| 6 | 0 | 0 |
| CLASS 11 | | |
| LAYER | UNRESOLVED | INDUCED |
| 1 | 0.102 | 0 |
| 2 | 0.003 | 0 |
| 3 | 0 | 0 |
| 4 | 0 | 0 |
| 5 | 0 | 0 |
| 6 | 0 | 0 |
| CLASS 12 | | |
| LAYER | UNRESOLVED | INDUCED |
| 1 | 0.054 | 0 |
| 2 | 0 | 0 |
| 3 | 0 | 0 |
| 4 | 0 | 0 |
| 5 | 0 | 0 |
| 6 | 0 | 0 |
| CLASS 13 | | |
| LAYER | UNRESOLVED | INDUCED |
| 1 | 0.063 | 0.0074 |
| 2 | 0.006 | 0.00267 |
| 3 | 0.002 | 0.00156 |
| 4 | 0 | 0.00144 |
| 5 | 0 | 0.001 |
| 6 | 0 | 0.00033 |

| | | |
|---|---|---|
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| CLASS 1 | | |
| LAYER | UNRESOLVED | INDUCED |
| 1 | 0.098 | 0.00067 |
| 2 | 0.005 | 0.00022 |
| 3 | 0 | 0.00033 |
| 4 | 0 | 0.00033 |
| 5 | 0 | 0.00022 |
| 6 | 0.001 | 0.00011 |
| CLASS 2 | | |
| LAYER | UNRESOLVED | INDUCED |
| 1 | 0.014 | 0.00011 |
| 2 | 0 | 0 |
| 3 | 0 | 0 |
| 4 | 0.001 | 0 |
| 5 | 0 | 0 |
| 6 | 0 | 0 |
| CLASS 3 | | |
| LAYER | UNRESOLVED | INDUCED |
| 1 | 0.047 | 0 |
| 2 | 0.003 | 0 |
| 3 | 0.003 | 0 |
| 4 | 0 | 0 |
| 5 | 0.001 | 0 |
| 6 | 0 | 0 |

**Table 9. V6.04 Sim Results (Concluded)**

| CLASS 14 | | |
|---|---|---|
| LAYER | UNRESOLVED | INDUCED |
| 1 | 0.029 | 0.00033 |
| 2 | 0.001 | 0.00056 |
| 3 | 0 | 0.00011 |
| 4 | 0.001 | 0 |
| 5 | 0 | 0 |
| 6 | 0 | 0.00011 |
| CLASS 15 | | |
| LAYER | UNRESOLVED | INDUCED |
| 1 | 0.036 | 0 |
| 2 | 0.002 | 0 |
| 3 | 0 | 0 |
| 4 | 0 | 0 |
| 5 | 0 | 0 |
| 6 | 0 | 0.00011 |
| CLASS 16 | | |
| LAYER | UNRESOLVED | INDUCED |
| 1 | 0.082 | 0.0044 |
| 2 | 0.016 | 0.00278 |
| 3 | 0.003 | 0.00133 |
| 4 | 0 | 0.00033 |
| 5 | 0 | 0 |
| 6 | 0 | 0.00022 |

| CLASS 4 | | |
|---|---|---|
| LAYER | UNRESOLVED | INDUCED |
| 1 | 0.028 | 0.00078 |
| 2 | 0.004 | 0.001 |
| 3 | 0.002 | 0.00022 |
| 4 | 0 | 0 |
| 5 | 0 | 0 |
| 6 | 0.001 | 0 |
| CLASS 5 | | |
| LAYER | UNRESOLVED | INDUCED |
| 1 | 0.019 | 0 |
| 2 | 0.003 | 0.00056 |
| 3 | 0.002 | 0 |
| 4 | 0 | 0.00011 |
| 5 | 0 | 0 |
| 6 | 0 | 0 |
| CLASS 6 | | |
| LAYER | UNRESOLVED | INDUCED |
| 1 | 0.069 | 0.00067 |
| 2 | 0.014 | 0 |
| 3 | 0.005 | 0.00011 |
| 4 | 0 | 0.00022 |
| 5 | 0 | 0 |
| 6 | 0 | 0 |

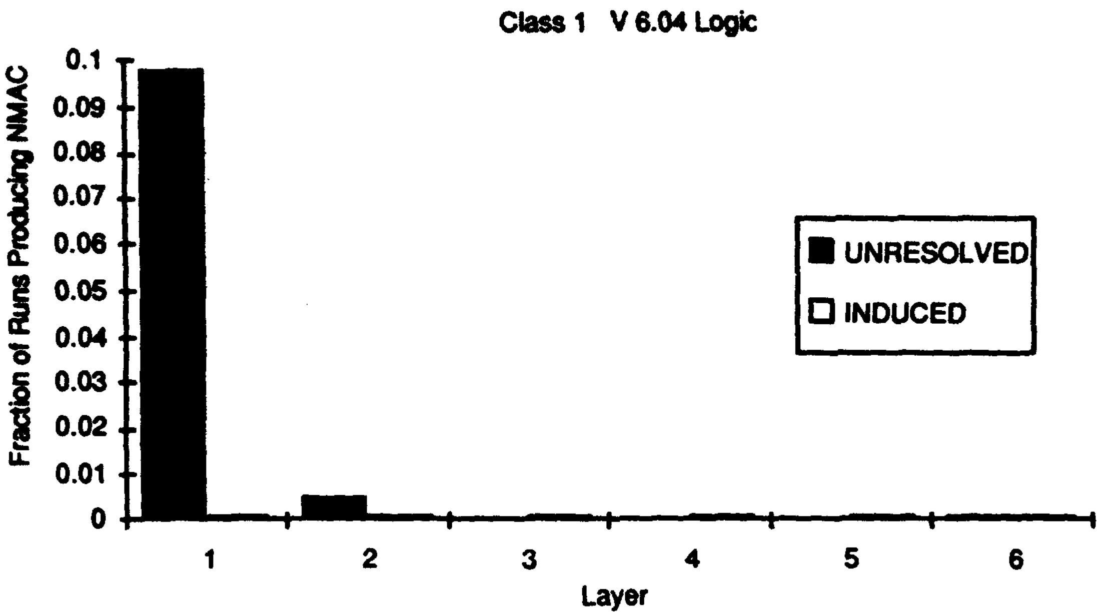


**Figure 29. Logic Performance—Class 1**

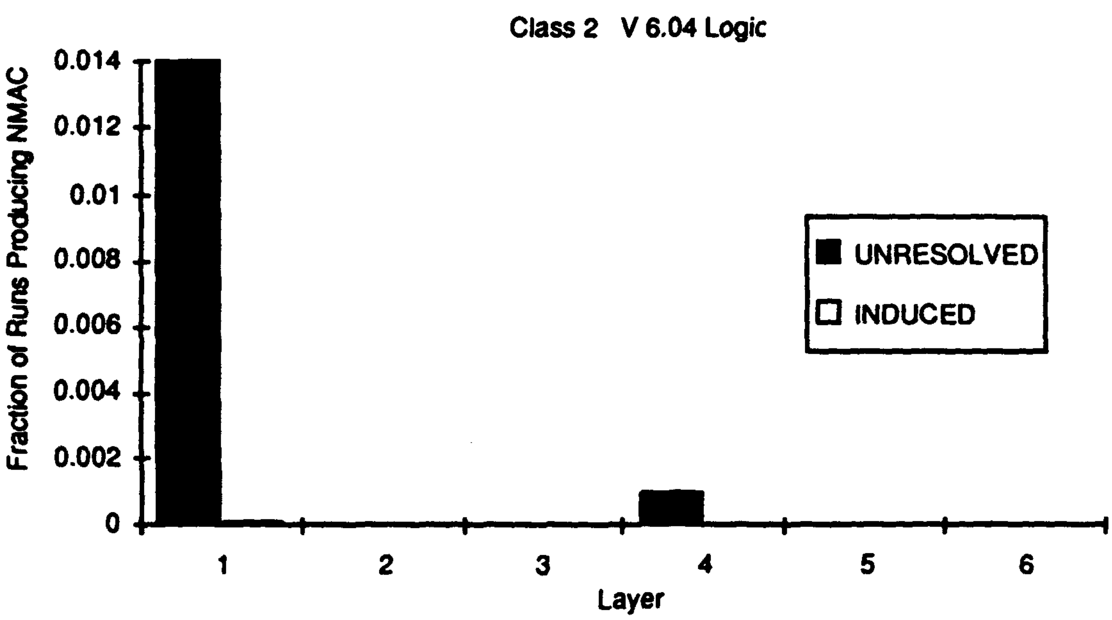


**Figure 30. Logic Performance—Class 2**

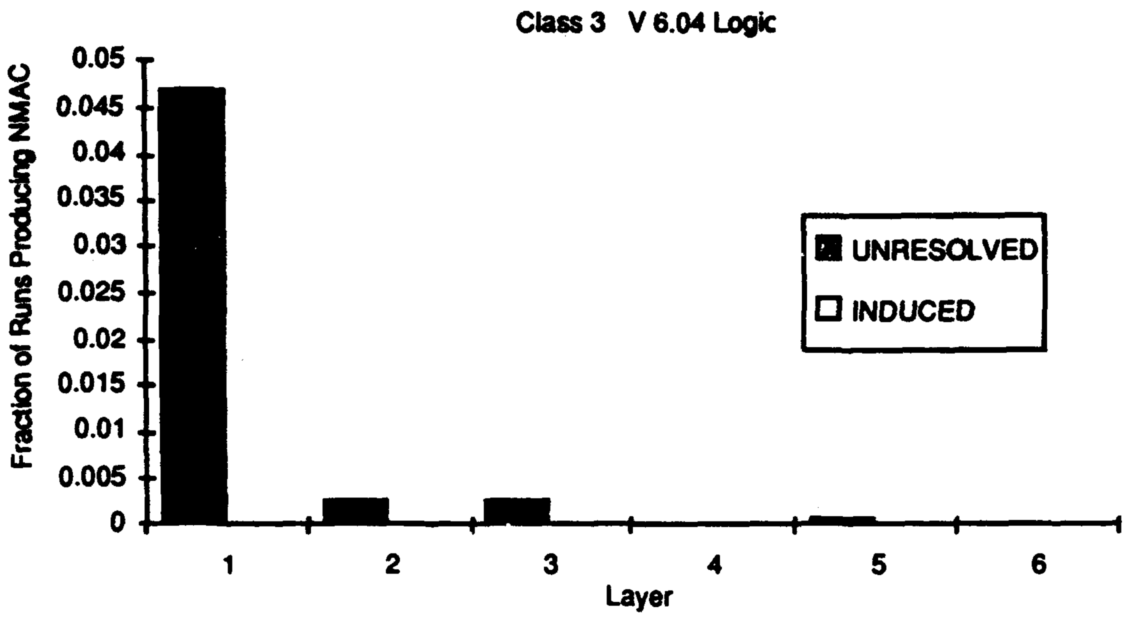


Figure 31. Logic Performance—Class 3

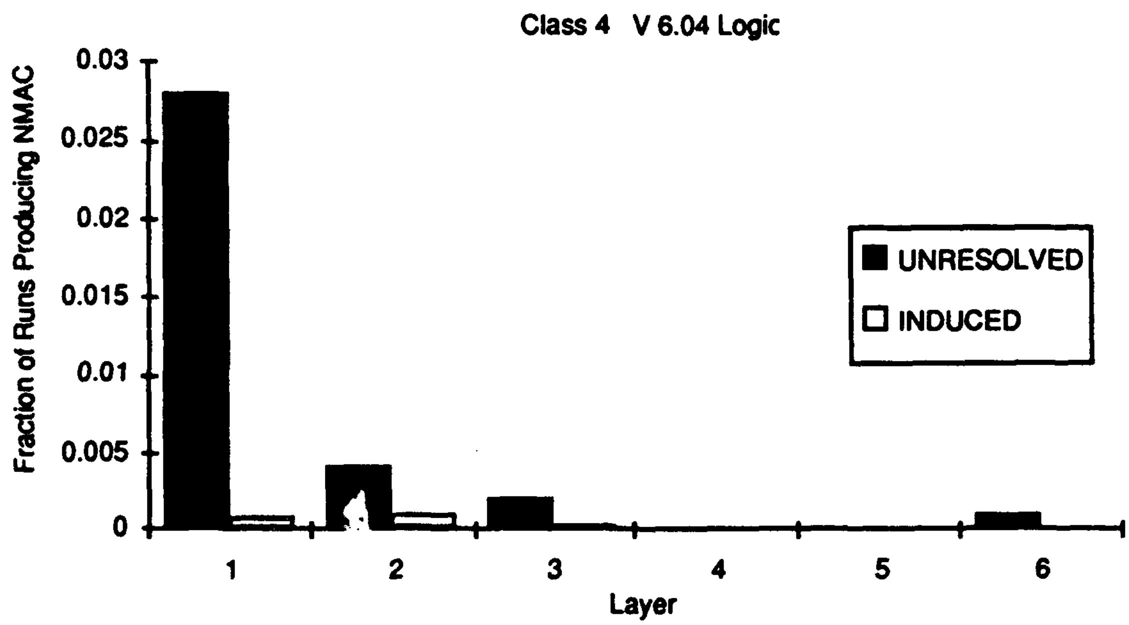


Figure 32. Logic Performance—Class 4

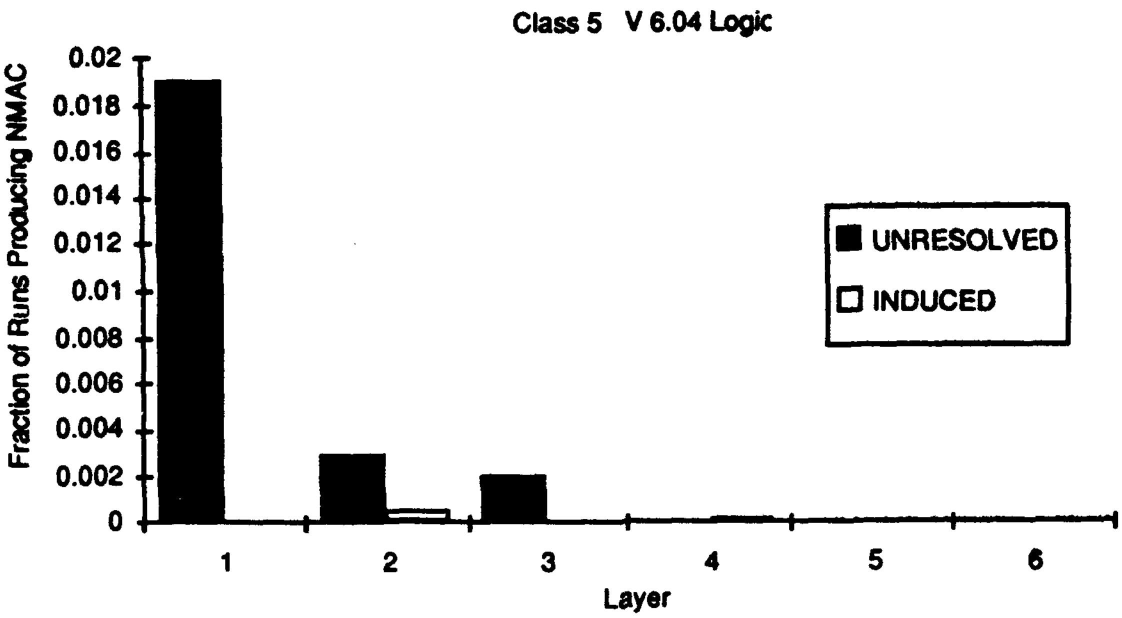


**Figure 33. Logic Performance—Class 5**

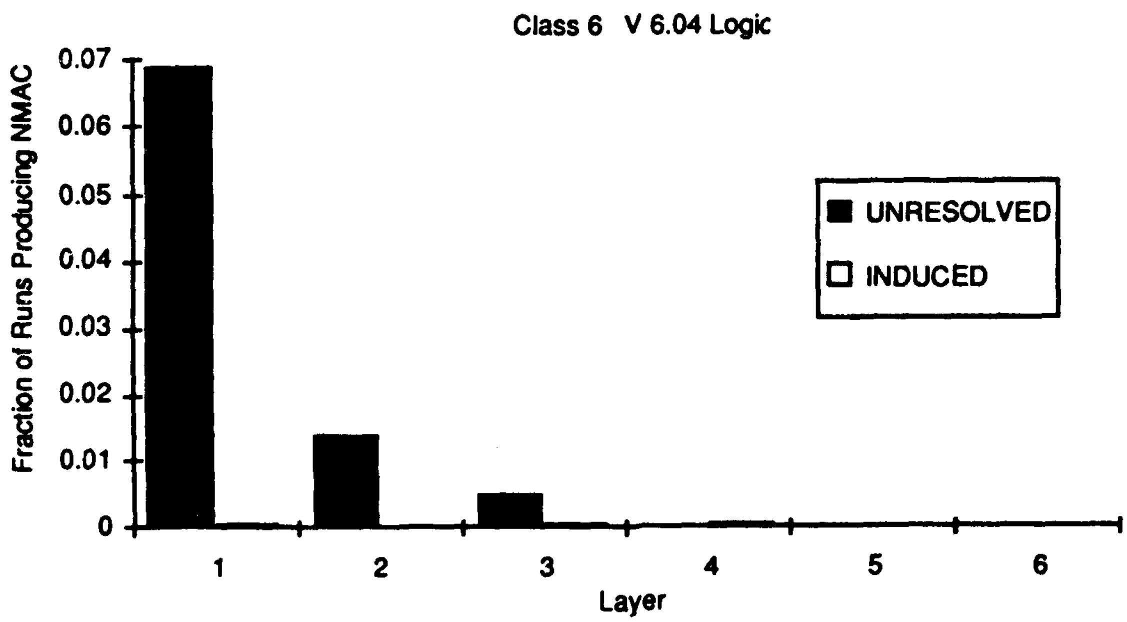


**Figure 34. Logic Performance—Class 6**

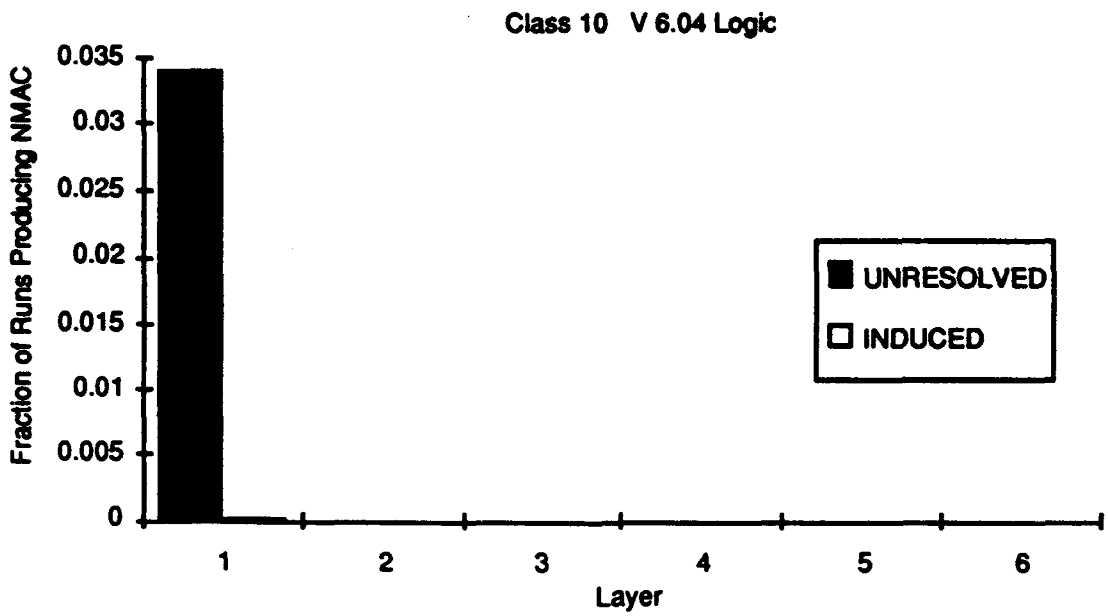


**Figure 35. Logic Performance—Class 10**

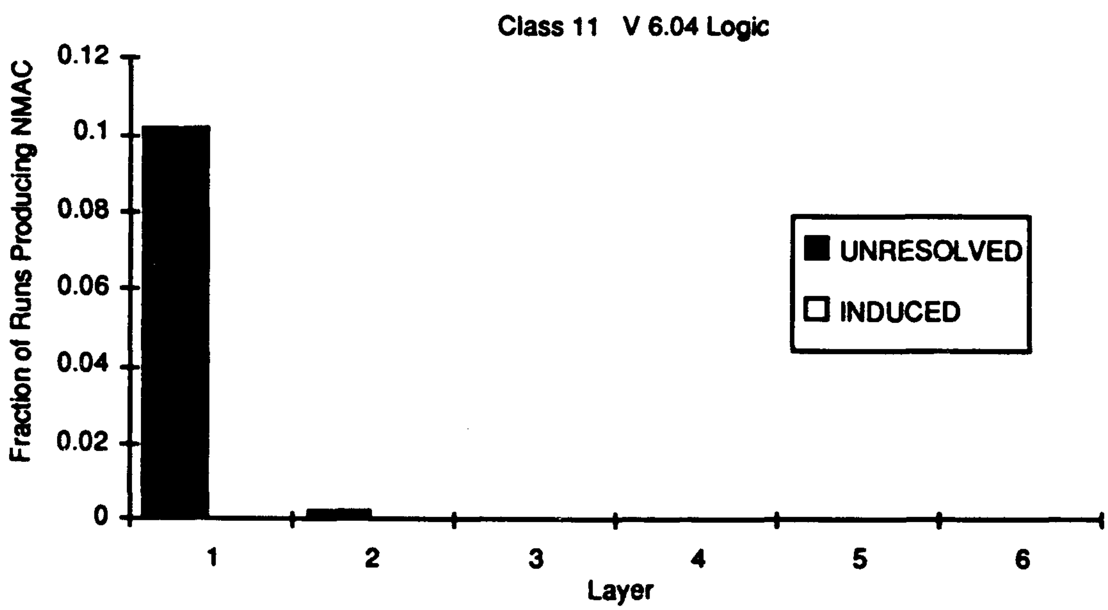


**Figure 36. Logic Performance—Class 11**

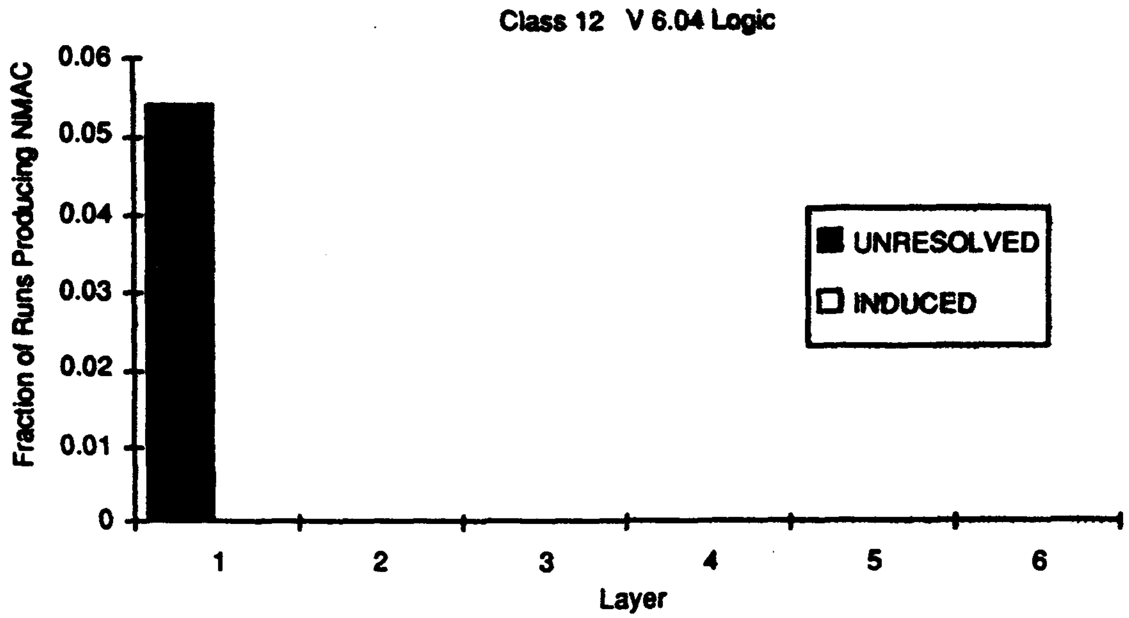


**Figure 37. Logic Performance—Class 12**

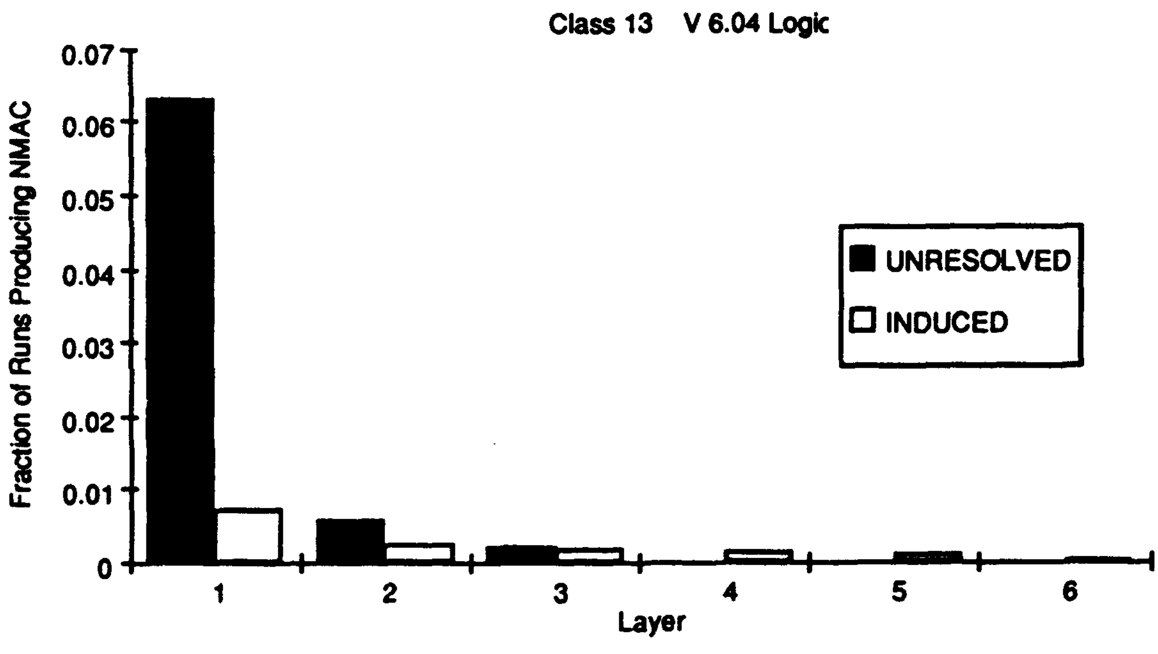


**Figure 38. Logic Performance—Class 13**

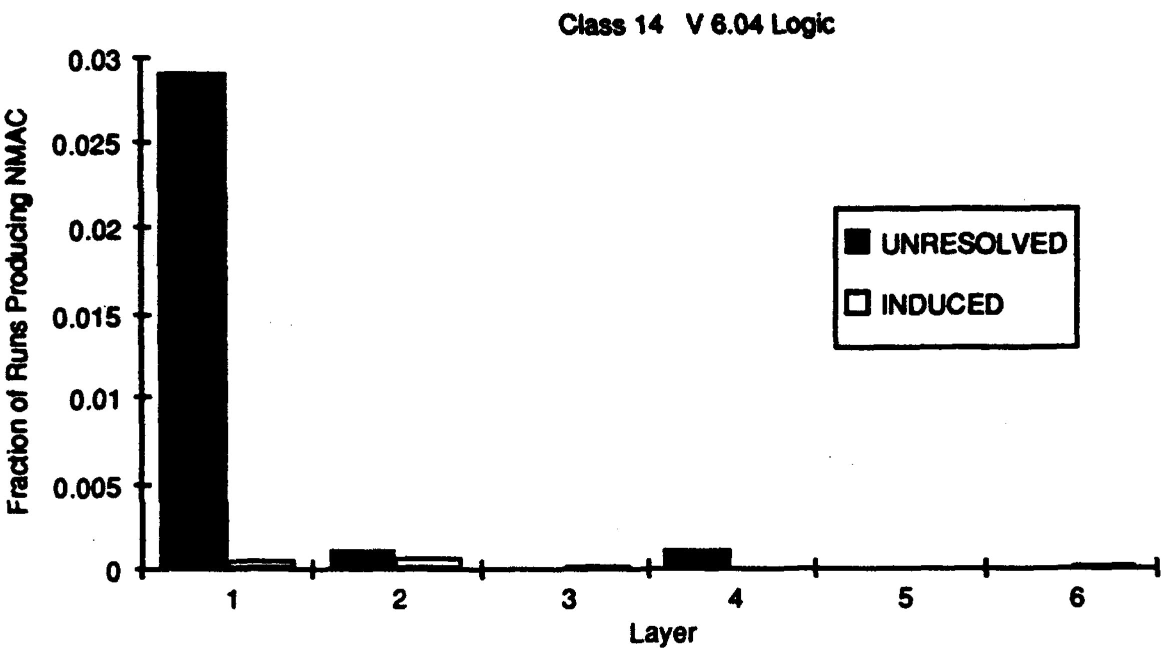


**Figure 39. Logic Performance—Class 14**

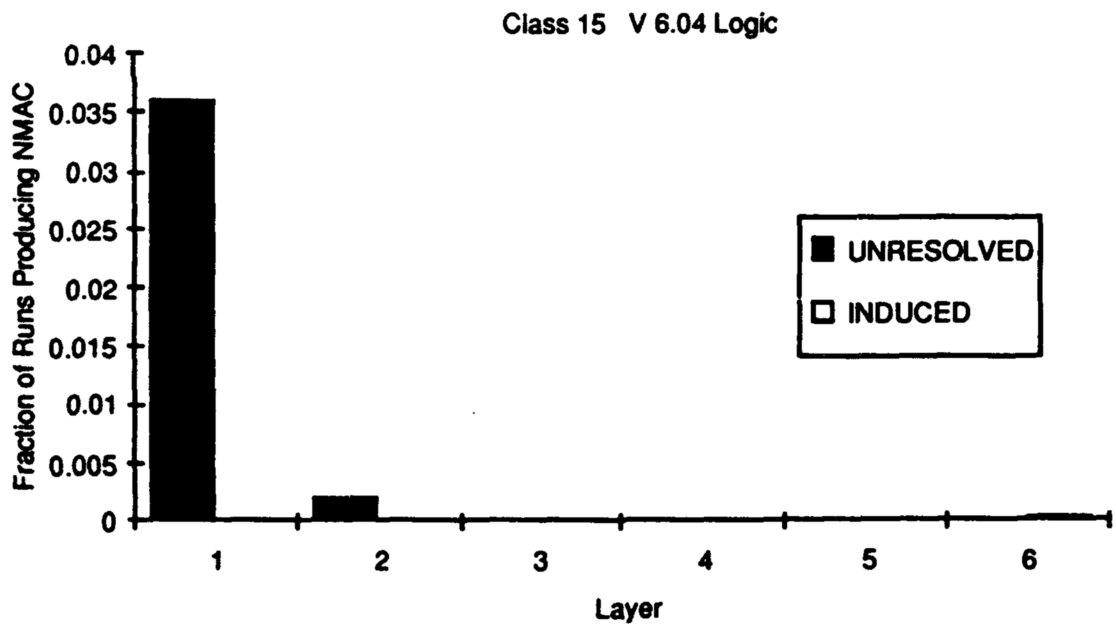


**Figure 40. Logic Performance—Class 15**

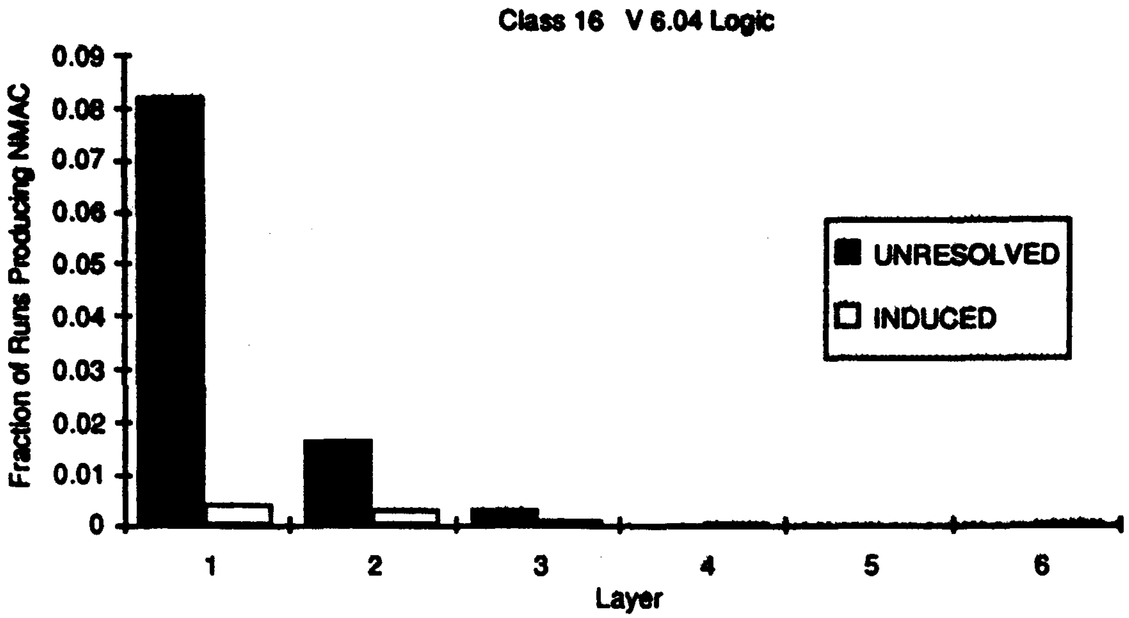


**Figure 41. Logic Performance—Class 16**

## APPENDIX D

## ALTIMETRY ERROR EFFECTS ON VERTICAL SEPARATION

This appendix addresses the effect of altimetry errors on the separation statistics of TCAS encounters generated by Monte Carlo simulations and provides formulas for incorporating these errors in a manner consistent with previous work.

A recent update to the TCAS II safety study [4] treats the net altimetry error, for any aircraft, as a random variable with a Laplacian (Double Exponential) distribution (equation 1) having one parameter, sigma, that is a function of altitude, altimeter maintenance and, possibly, other factors. Empirical values for sigma have been reported for seven altitude regions.

$$f(x) = (2\,\sigma)^{-1} \exp\left(\frac{-|x|}{\sigma}\right) \qquad 1.$$

The vertical *overlap density* for a pair of aircraft is given by the convolution of two such Laplacians. Therefore, the probability that two aircraft with a reported vertical separation, S, are actually within h feet of each other can be found by integrating the convolution over the vertical interval S ± h. [For instance, the vertical NMAC region would have h = 100 feet.] The following treats separately the cases in which the two parameters are equal and unequal.

When the parameters are **equal**, the convolution is given by equation 2 [9].

$$C(z) = (4\,\sigma^2)^{-1}\,(|z| + \sigma) \exp\left(\frac{-|z|}{\sigma}\right) \qquad 2.$$

The formulas for computing the overlap probability are given below. Since the convolution contains an absolute value, there are two cases to consider, viz., $(S - h) \geq 0$ and $(S - h) \leq 0$ where S and h are taken as positive quantities.

For $(S - h) \geq 0$, the overlap probability is given by equation 3.

$$\mathrm{Prob}(|sep| \leq h) = (4\,\sigma)^{-1} \exp\left(\frac{-(S + h)}{\sigma}\right) \left[\exp\left(\frac{2\,h}{\sigma}\right)(2\,\sigma + S - h) - (2\,\sigma + S + h)\right] \qquad 3.$$

For $(S - h) \leq 0$, the overlap probability is given by equation 4.

$$\mathrm{Prob}(|sep| \leq h) = 1 - (4\,\sigma)^{-1} \exp\left(\frac{-(S + h)}{\sigma}\right) \left[\exp\left(\frac{2\,S}{\sigma}\right)(2\,\sigma - S + h) + (2\,\sigma + S + h)\right] \qquad 4.$$

When the parameters are **unequal**, the convolution is given by equation 5.

$$C(z) = \frac{\sigma_1 \exp(\frac{-|z|}{\sigma_1}) - \sigma_2 \exp(\frac{-|z|}{\sigma_2})}{2\,(\sigma_1^2 - \sigma_2^2)} \qquad 5.$$

Once again, there are two cases:

For (S - h) $\geq$ 0, the overlap probability is given by equation 6.

$$\mathrm{Prob}(|sep| \leq h) = \frac{\sigma_1^2 \exp(\frac{-S}{\sigma_1}) \sinh(\frac{h}{\sigma_1}) - \sigma_2^2 \exp(\frac{-S}{\sigma_2}) \sinh(\frac{h}{\sigma_2})}{\sigma_1^2 - \sigma_2^2} \qquad 6.$$

For (S - h) $\leq$ 0, the overlap probability is given by equation 7.

$$\mathrm{Prob}(|sep| \leq h) = \frac{\sigma_1^2\,[1 - \exp(\frac{-h}{\sigma_1}) \cosh(\frac{S}{\sigma_1})] - \sigma_2^2\,[1 - \exp(\frac{-h}{\sigma_2}) \cosh(\frac{S}{\sigma_2})]}{\sigma_1^2 - \sigma_2^2} \qquad 7.$$

Note that, when S = h, equations 3 and 4 become equivalent as do equations 6 and 7.

These equations were further checked by comparison with the results of Monte Carlo simulations. Five million iterations were carried out for each of the four cases (eqs. 3, 4, 6 and 7). Variance reduction was achieved through the use of antithetic variates [A-1]. The results are presented in table 10.

**Table 10. Simulation versus Analytical Results**

| Equation | S | h | $\sigma_1$ | $\sigma_2$ | Prob($\lvert sep \rvert \leq$ h) | |
|---|---|---|---|---|---|---|
| | | | | | Simulation | Equation |
| 3 | 1000 | 100 | 144 | 144 | 0.002823 | 0.002822 |
| 4 | 200 | 300 | 165 | 165 | 0.583963 | 0.583859 |
| 6 | 500 | 100 | 100 | 150 | 0.039614 | 0.039716 |
| 7 | 200 | 500 | 100.0 | 100.1 | 0.935710 | 0.935633 |

## LIST OF REFERENCES

A-1 Rubinstein, R., 1981, *Simulation and the Monte Carlo Method*, pp. 135-138, John Wiley and Sons, New York.

# GLOSSARY

| | |
|---|---|
| AGL | Above Ground Level |
| ARTS | Automated Radar Terminal System |
| ATC | Air Traffic Control |
| CPA | Closest Point of Approach |
| FAA | Federal Aviation Administration |
| FPM | Feet Per Minute |
| GA | General Aviation |
| HMD | Horizontal Miss Distance |
| IFR | Instrument Flight Rules |
| NMAC | Near Midair Collision |
| NMI | Nautical Mile |
| RA | Resolution Advisory |
| TA | Traffic Advisory |
| TCAS | Traffic Alert and Collision Avoidance System II |
| VFR | Visual Flight Rules |
| VMD | Vertical Miss Distance |